Optimal Portfolio Choice with Annuities and Life Insurance for Retired Couples*

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Abstract. Using a portfolio choice model, we derive the optimal demand for stocks, bonds, annuities, and term life insurance for a retired couple with uncertainty in both lifetimes. We show that the optimal portfolio is heavily weighted with joint annuities and that life insurance is purchased mainly to protect a surviving spouse from loss of annuitized income rather than for bequest. Consistent with these predictions, empirical analyses on Health and Retirement Study data indicate that life insurance holdings are related to the degree of asymmetry in the couple’s annuitized income distribution.

JEL Classification: G11, G22, D14, D91

1. Introduction

Couples in retirement face the challenge to derive consumption and investment policies that will maximize their joint lifetime utility, subject to preexisting retirement income and savings accumulated during the worklife. The investment menu for the retirement nest-egg may include capital market instruments, typically stocks and bonds, or insurance products, such as term life insurance or life annuity contracts. As of now, it is an open question how retired couples should optimally combine these products in their retirement portfolio. The majority of the papers in the lifecycle literature analyze optimal portfolio strategies either for single individuals or, when looking at households, for single representative agents.1 Variation in family size, however, is in general not modeled explicitly.

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1 See, for instance, Cocco, Gomes, and Maenhout (2005).
Consequently, these models are unable to describe the effects of a death of one spouse on the financial position and consumption choice of the surviving partner.

The contribution of this article is to shed more light on the joint financial decision making of retired couples, by developing a realistically calibrated dynamic two-person lifecycle consumption and portfolio choice model that explicitly accounts for uncertainty in both lifetimes. This model allows us to derive a couple’s optimal demand for risk-free bonds, risky stocks, (single and joint) annuities, and term life insurance policies. Moreover, the model enables us to answer the question as to what is the key driver of life insurance demand, “a provision motive or a pure bequest motive” (Modigliani, 1988; Hurd, 1999). According to the former, term life insurance is purchased to ensure that a surviving spouse will be able to afford an adequate level of consumption after the partner’s death and the resulting loss of his or her life-contingent income. According to the latter, life insurance is purchased to support the couple in bequeathing wealth to the next generation. In this sense, our article also contributes to the literature on the role of the bequest motive in lifecycle portfolio choice. Bernheim (1991) and, more recently, Inkmann and Michaelides (2012), argue that the empirically observed life insurance demand of households in the USA and the UK indicates the existence of a bequest motive. Yet, both papers base their analysis on a model that does not allow for uncertainty in family size. Hence, both are unable to disentangle the provision motive-related demand for life insurance from that driven by the pure bequest motive.

We find that couples seek to build up retirement portfolios with survival-contingent income streams that are balanced between the two partners. A key driver is the consumption scaling factor, which describes the change in consumption required to maintain a household’s utility level upon a change in family size. Moreover, we show that the demand for term life insurance is primarily driven by the provision motive and only to a much lesser extent by a pure bequest motive. In particular, if a large fraction of retirement wealth is pre-annuitized for only one spouse, life insurance contracts will be purchased to insure against the loss of annuity income due to the early death of the better endowed partner. This result is valid irrespective of whether the couple has a pure bequest motive or not. If one spouse dies, the surviving partner has almost no demand for term life insurance even in the presence of a bequest motive. Intended intergenerational wealth transfer

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is preferably financed by a balanced portfolio of stocks and bonds. Only couples with relatively low liquid wealth at retirement will purchase term life insurance to provide for bequests.

While our analysis is primarily normative, we show that our model predictions are also consistent with empirical evidence. Using data from the US Health and Retirement Study (HRS), we show that, for most retired couples, annuitized pension income from sources other than Social Security is asymmetrically distributed between husbands and wives. We also show that husbands are more likely to hold term life insurance than wives, and that they hold much higher insurance face values. Econometric analyses indicate that husbands’ higher participation rates as well as face values are statistically significantly related to the degree of asymmetry in the couple’s pension income distribution. By contrast, the number of children has no significant impact on a retired couple’s life insurance demand. Both results are in line with the prediction of our theoretical model and support the hypothesis that term life insurance demand of retired couples is mostly driven by a provision motive and not by a pure bequest motive.

According to the HRS, old age insurance benefits from Social Security are the major source of retirement income for most retired households. In practice, these benefits, which are comparable to a joint and survivor annuity, are well balanced for a couple and provide relatively generous survivor benefits in the range of one-half to two-thirds of the previous income. Against this background, we also show that, for retired couples with moderate financial wealth, purchasing additional annuities in the private market only provides marginal welfare gains. This might explain why so few households participate in the private annuity market and therefore—at least in part—the annuity puzzle.

This article builds on and extends previous research on the impact of longevity risk on lifecycle portfolio choice in several directions. Kotlikoff and Spivak (1981) show that mortality risk pooling in multi-individual households leads to welfare gains comparable to having access to the annuity market. Auerbach and Kotlikoff (1991) discuss the role of life insurance in protecting a sustainable consumption level for widows, and they show that the life insurance holdings for many US couples are too low. Brown and Poterba (2000) study couples’ welfare gains from having access to joint life annuity products, based on the annuity equivalent wealth framework developed by Mitchell et al. (1999). Their findings suggest that these gains can be as high as 70% of the retirement wealth, depending on

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3 Here and in the following, we use the term Social Security to refer only to old age insurance benefits and not to other system provisions, such as disability benefits.
preference parameters and the amount of preexisting annuity income. However, their assumptions are quite restrictive: the model setting only allows for complete annuitization of all accumulated assets at the beginning of the retirement phase, given an exogenously specified survivor benefit factor. Moreover, they do not allow for investments in risky stocks or life insurance contracts, and they only compare welfare gains of an annuitization strategy against a benchmark of riskless bonds.

Studies by Horneff, Maurer, and Stamos (2008b) and Horneff et al. (2009, 2010) integrate individual annuities into realistically calibrated lifecycle portfolio choice models, allowing for risky stock investments and optimal gradual annuitization strategies. They demonstrate that these products are welfare-enhancing, in that they offer consumers an effective hedge against individual longevity risk as well as the opportunity to trade liquidity for the survival-contingent extra return known as the “survival credit”. Yet, they model a single household rather than a couple. Moreover, they do not include term life insurance in their investment universe. More recently, Love (2010) studies shocks in family size within a dynamic lifecycle model. While that study allows for term life insurance purchases, single and joint annuities are neglected. Inkmann, Lopez, and Michaelides (2011) simultaneously study the demand for payout annuities and life insurance within a lifecycle model of consumption and portfolio choice. They show that their model, if appropriately calibrated, is able to explain the low annuity market participation observable in their rich data set for UK households. Yet, that study takes the perspective of a single representative household and does not account for variations in the number of family members.5

In what follows, we present the structure of our lifecycle model for a retired couple with uncertain individual lifetimes, which has access to the various investment and insurance products to manage their consumption and bequest requirements. We then explore and discuss the role of these products in optimal lifecycle profiles. Subsequently, we compare our model predictions to empirical evidence from the HRS. A final section concludes.

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4 The survival credit is the excess return that surviving annuitants receive as the insurance company redistributes the funds of the deceased members of the annuity pool.

5 Chen et al. (2006), extending earlier work by Campbell (1980), also study life insurance in a portfolio context with uncertain labor income. Yet, their analysis is based on a static model for a representative single household that does not account for annuities.
2. The Lifecycle Optimization Model for a Retired Couple

2.1 FAMILY DYNAMICS

We model a couple in retirement that faces an uncertain lifespan governed by sex-specific 1-year survival probabilities, $p^x_t$ and $p^y_t$. Here and in the following, $x$ ($y$) represents the husband (wife) as well as his (her) age and $t = (0, \ldots, T)$ the time after the couple retired (measured in years). We assume that the survival probabilities are independent, such that the demise of one spouse does not affect the survival probability of the other spouse.

At each point in time, the couple may be in either one of the four different family states $s_t$: both being alive, wife having deceased (widower), husband having deceased (widow), and both having deceased. Using indicator variables $I^x_t$ and $I^y_t$, which are one if the husband/wife is alive at time $t$ and zero otherwise, these four states can be represented by

$$
\begin{align*}
1 : & \quad I^x_t = 1, \quad I^y_t = 1 \quad \text{both alive} \\
2 : & \quad I^x_t = 1, \quad I^y_t = 0 \quad \text{widower} \\
3 : & \quad I^x_t = 0, \quad I^y_t = 1 \quad \text{widow} \\
4 : & \quad I^x_t = 0, \quad I^y_t = 0 \quad \text{both deceased.}
\end{align*}
$$

The time-dependent transition matrix $\Pi_{ij,t} = \text{Prob}(s_{t+1} = i \mid s_t = j)$ of this Markov chain is specified by the individual 1-year survival probabilities:

$$
\Pi_t = \begin{pmatrix}
0 & 0 & 0 \\
(p^x_t \cdot p^y_t) & 0 & 0 \\
(1 - p^x_t) \cdot p^y_t & p^y_t & 0 \\
(1 - p^x_t)(1 - p^y_t) & 1 - p^x_t & 1 - p^y_t
\end{pmatrix}.
$$

At the end of our projection horizon $T$, we set $p^x_T = p^y_T = 0$. We neglect all changes in the family state other than those driven by mortality, such as a divorce or a new partnership after one of the spouses has died.

2.2 FINANCIAL PRODUCTS

The couple can select among different financial products to manage retirement income and potential bequests: riskless bonds, risky stocks, term life insurances, and annuities. Bonds have a constant annual real gross rate of return, represented by $R_f$. The annual real gross return on stocks, $R_s$, is serially independent and identically log normally distributed.
In each period \( t \), a 1-year term life insurance contract can be purchased for each living spouse \( i \in \{x, y\} \). If the insured spouse dies within the period \([t, t + 1]\), the insurer will pay the face value \( L_i^t \) at time \( t + 1 \). Since the term life insurance does not build up cash value, the payment is zero in case the insured spouse survives. Based on the actuarial principle of equivalence, the insurance premium \( LP_i^t \) is equal to the present value of the expected payout. Assuming that the insurance company will charge an expense loading of \( \lambda_{LI} \), the premium is calculated according to:

\[
LP_i^t = (1 + \lambda_{LI}) \cdot (1 - p_i^t) \cdot \frac{L_i^t}{R_f^t}. \tag{3}
\]

As an investment, term life insurance provides only moderate expected returns, but it exposes investors to enormous return volatilities, due to very high payout in case of death and a total loss in case of survival. At the same time, life insurance payouts are, by construction, negatively correlated with annuitized income, such as Social Security, company pensions, and private annuities.

Similar to life insurance, single annuities can be purchased for each individual spouse. Let \( \tilde{A}_i^t \) denote a normalized annuity that pays a constant amount of one monetary unit per year as long as the annuitant \( i \in \{x, y\} \) is alive. The sex- and age-dependent annuity factor \( \tilde{a}_i^t \) is the nonrefundable premium for \( \tilde{A}_i^t \) charged by the annuity provider at time \( t \). Accounting for a loading factor \( \lambda_A \), \( \tilde{a}_i^t \) is given by

\[
\tilde{a}_i^t = (1 + \lambda_A) \sum_{\tau=t+1}^{T} \frac{p_{\tau,t}^i}{(R_f)^{\tau-t}}. \tag{4}
\]

Here, \( p_{\tau,t}^i \) denotes the probability that spouse \( i \) is alive at time \( \tau \) conditional on being alive at time \( t \), which is the product \( p_{\tau,t}^i = \prod_{s=t}^{\tau-1} p_s^i \) of the 1-year survival probabilities.

\[\text{From the policyholder's perspective, the unconditional expected one-period return of a life insurance contract is given by } E(R_{LI}^t) = \frac{R_f}{1+\lambda_{LI}} \text{ and the corresponding return volatility is } STD(R_{LI}^t) = \frac{R_f}{1+\lambda_{LI}} \sqrt{\frac{1}{1-p}}. \]

To gain economic intuition, let us assume that the risk-free rate is \( R_f = 1.02 \), the one-period mortality is \( 1 - p_i^t = 10\% \), and that the insurer charges an actuarially fair premium, i.e., \( \lambda_{LI} = 0 \). Then, the unconditional expected gross return is 102\%. The return volatility is 306\%, resulting from a gross return of 1,020\% in case of death and −100\% in case of survival.
A normalized joint annuity $\tilde{A}^{xy}$ pays one monetary unit per year as long as at least one of the spouses is alive. The corresponding annuity factor is given by:

$$\tilde{a}_t^{xy} = (1 + \lambda_A) \sum_{\tau = t+1}^{T} \frac{p_{\tau,t}^{xy}}{(R_f)^{T-\tau}},$$

(5)

where $p_{\tau,t}^{xy}$ denotes the probability that at least one of the spouses is alive at time $\tau$, conditional on both being alive at time $t$:

$$p_{\tau,t}^{xy} = p_{\tau,t}^x p_{\tau,t}^y + p_{\tau,t}^x (1 - p_{\tau,t}^y) + (1 - p_{\tau,t}^x) p_{\tau,t}^y = p_{\tau,t}^x + p_{\tau,t}^y - p_{\tau,t}^x p_{\tau,t}^y.$$

(6)

Joint annuities do not necessarily pay a constant amount until the last spouse dies. Typically, a survivor benefit ratio $K \leq 1$ is chosen to which the payment level is reduced upon the first death. We call these annuities “$K\%$ joint annuities” and denote them $\tilde{A}^{xy}(K)$. We do not have to explicitly include these annuities in our model, since any survivor benefit ratio $K$ can be replicated by linearly combining the three elementary annuities $\tilde{A}^x$, $\tilde{A}^y$, and $\tilde{A}^{xy}$ according to

$$\tilde{A}^{xy}(K) = (1 - K)(\tilde{A}^x + \tilde{A}^y) + (2K - 1)\tilde{A}^{xy}.$$  

(7)

Insurance companies use special life tables to calculate the premiums for life insurance and annuity products. Therefore, the survival probabilities used in pricing, i.e., in Equations (3–5), need not be identical to those governing the family Markov process in Equation (2). In this case, loadings charged by insurance companies may consist of two parts: explicit loadings to cover administrative costs or sales commissions ($\lambda_{A1}$ and $\lambda_A$) and implied loadings resulting from asymmetry in the mortality beliefs of the insurance company and the policyholder.

2.3 WEALTH DYNAMICS

In each period, the household must decide how much of its liquid wealth ($W_t$) to spend on consumption ($C_t$), life insurance premiums ($LP^x_t$, $LP^y_t$), and annuity premiums ($AP^x_t$, $AP^y_t$, $AP^{xy}_t$). Moreover, the household must choose the fraction of the remaining wealth to invest in stocks ($\pi_t$). Next period’s liquid wealth is given by the remaining wealth, including capital market
returns, annuity income \((A^x_t, A^y_t, A^{xy}_t)\), and, if one of the spouses has died, payments from life insurance contracts \((L^x_t, L^y_t)\):

\[
W_{t+1} = \left( W_t - C_t - LP^x_t - LP^y_t - AP^x_t - AP^y_t - AP^{xy}_t \right) \cdot \left( R_f + \pi_t \cdot (R_{t+1} - R_f) \right) + \dot{A}^x_{t+1} + \dot{A}^y_{t+1} + \dot{A}^{xy}_{t+1} + \left( 1 - \Pi^x_{t+1} \right) L^x_t + \left( 1 - \Pi^y_{t+1} \right) L^y_t.
\]  

(8)

The dynamics of the annuity payments \(A^x, A^y,\) and \(A^{xy}\) are given by

\[
\dot{A}^x_{t+1} = \Pi^x_{t+1} \left( A^x_t + \frac{AP^x_t}{\bar{a}^x_t} \right)
\]  

(9)

\[
\dot{A}^y_{t+1} = \Pi^y_{t+1} \left( A^y_t + \frac{AP^y_t}{\bar{a}^y_t} \right)
\]  

(10)

\[
\dot{A}^{xy}_{t+1} = \left( \Pi^x_{t+1} + \Pi^y_{t+1} - \Pi^{xy}_{t+1} \Pi^y_{t+1} \right) \left( A^{xy}_t + \frac{AP^{xy}_t}{\bar{a}^{xy}_t} \right).
\]  

(11)

We impose that the household is liquidity constrained, such that money cannot be borrowed for financing consumption, insurance products, or stock investments. Furthermore, we disallow short positions in stocks, life insurance, and annuity products:

\[
W_t - C_t - LP^x_t - LP^y_t - AP^x_t - AP^y_t - AP^{xy}_t \geq 0
\]  

(12)

\[
0 \leq \pi_t \leq 1 \quad LP^x_t \geq 0 \quad LP^y_t \geq 0
\]  

(13)

\[
AP^x_t \geq 0 \quad AP^y_t \geq 0 \quad AP^{xy}_t \geq 0.
\]  

(14)

Annuities are illiquid in the sense that premiums are nonrefundable and that households are generally prevented from selling them on a secondary market. Accordingly, previously purchased annuity income cannot be reduced, and the range of attainable survivor benefit ratios in the embedded \(K\%\) joint annuities is restricted to \(0.5 \leq K \leq 1\). Finally, it is self-evident that we only allow for purchases of life insurance and annuities products by the living. So, for example, a widow may still buy single annuities for herself but not joint annuities.

2.4 OPTIMIZATION PROBLEM

The household draws utility from consumption according to a time-additive utility function of the constant relative risk aversion type:

\[
u(C_t, s_t) = \frac{1}{1 - \gamma} \left( \frac{C_t}{\bar{s}_t} \right)^{1-\gamma},
\]  

(15)
where $\gamma$ represents the level of relative risk aversion. Following Love (2010), we normalize the total consumption by the scaling factor $C_{13}$, which depends on the family state $s_t$. This specification assumes that consumption is equally shared between the spouses, $C_x = C_y = C/2$, and that there is jointness in consumption. This is equivalent to the approach by Brown and Poterba (2000) under the assumption of equally weighted, identical subutility functions. This factor can be interpreted as the effective family size. By definition, it is equal to one for a single person household. For a multiperson household, it indicates how joint consumption must be increased to gain the same utility as a single person. Through shared consumption and economies of scale, this factor is expected to be below the number of family members.

The household seeks to maximize expected lifetime utility, expressed recursively through the Bellman equation

$$J_t(W_t, A_t, s_t) = \max_{C_t, \pi_t, AP_t, LP_t} \left\{ \frac{1}{1 - \gamma} \left( \frac{C_t}{\phi_{s_t}} \right)^{1-\gamma} + \beta E_t[J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1})] \right\},$$

(16)

where $\beta$ represents the time preference rate. The value function is governed by the state variables liquid wealth $W_t$, the vector of annuity payments $A_t = (A_t^x, A_t^y, A_t^{xy})'$, and the family state $s_t$. The controls are consumption $C_t$, asset allocation $\pi_t$, premiums for annuity purchases $AP_t$, and premiums for life insurance purchases $LP_t$.

If both spouses are alive ($s_t = 1$), the expectation term inside the value function is given by:

$$E_t[J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1})] = p^x_t p^y_t E_t[J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1} = 1)]$$

$$+ p^x_t (1 - p^y_t) E_t[J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1} = 2)]$$

$$+ (1 - p^x_t) p^y_t E_t[J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1} = 3)]$$

$$+ (1 - p^x_t)(1 - p^y_t) E_t[J_{t+1}(W_{t+1}, A_{t+1}, s_{t+1} = 4)].$$

(17)

The expectations on the right-hand side are conditioned on the family state $s_{t+1}$ and the associated future value functions are weighted only by corresponding survival probabilities. Hence, we do not consider one of the spouses to be a decision maker, who puts more weight on her/his value function. For widowers and widows ($s_t = \{2, 3\}$), the expectation only includes one term for the survival of the remaining spouse and one in which the last spouse has deceased, weighted in accordance with the family state transition matrix (2).
If both spouses have died \((s_t = 4)\), the value function depends on the couple’s (pure) bequest motive, i.e., the direct utility from leaving wealth to the next generation. Following Gomes and Michaelides (2005) and Love (2010), the form of the bequest function is given by:

\[
J_t(W_t, A_t, s_t = 4) = \frac{1}{1 - \gamma} \left( \frac{W_t}{B} \right)^{1-\gamma} \quad \text{for } B > 0
\]

\[
J_t(W_t, A_t, s_t = 4) = 0 \quad \text{for } B = 0.
\]

The strength of the bequest motive is measured by the parameter \(B\). As pointed out by Ameriks et al. (2011), the bequest parameter in this functional form can be interpreted as the couple’s wish to leave an inheritance of about \(B/\phi\) times, the annual consumption in order smooth utility over time.

At time \(T\), the survival probabilities are zero and the terminal conditions are given by:

\[
J_{T+1} = \frac{1}{1 - \gamma} \left( \frac{W_{T+1}}{B} \right)^{1-\gamma} \quad \text{for } B > 0
\]

\[
J_T = \frac{1}{1 - \gamma} \left( \frac{W_T}{\phi_{ST}} \right)^{1-\gamma} \quad \text{for } B = 0.
\]

The optimization problem is homothetic in wealth. Hence, we can decrease the computational effort by normalizing annuity payments by liquid wealth and thereby reduce the number of continuous state variables. The value function then takes the form

\[
J_t(W_t, A_t, s_t) = (W_t)^{1-\gamma} j(a_t, s_t)
\]

with the normalized annuity vector \(a_t = (a_t^x, a_t^y, a_t^{xy})' = (A_t^x/W_t, A_t^y/W_t, A_t^{xy}/W_t)\). Instead of directly working with the three continuous state variables \((a_t^x, a_t^y, a_t^{xy})\), we discretize the normalized total amount of annuity payments \(\bar{a}_t = a_t^x + a_t^y + a_t^{xy}\), the share of joint annuities \(a_t^{xy}/\bar{a}_t\), and the husband’s share of single annuities \(a_t^x/(a_t^x + a_t^y)\) on a \(20 \times 19 \times 19\) grid. This choice has the advantage that all continuous state variables lie in the interval \([0, 1]\) and all their combinations are attainable. While \(a_t^x, a_t^y, \) and \(a_t^{xy}\) lie in \([0, 1]\) as well as many combinations result in

\[7\] Love (2010) relates the strength of the bequest parameter to the number of children. Other studies, for instance, Ameriks et al. (2011), extend the bequest function (20) by incorporating an additional parameter to measure the degree to which bequest is a luxury good. To keep the model parsimonious, we refrain from incorporating this additional parameter.
\( \ddot{a}_t > 1 \), i.e., \( A_t > W_t \), which violates the budget restriction (8). Hence, our approach avoids unnecessary computational effort while preserving the rectangular grid, which facilitates interpolation.

A three-dimensional grid for annuity payments is only necessary as long as both spouses are alive \( (s_t = 1) \). For a widow/widower \( (s_t = \{2, 3\}) \), it is sufficient to only track total annuity payments, which we discretize on a \( 20 \times 1 \) grid. For every grid point, we solve for optimal control variables by evaluating the expectation of the future value function using Gaussian quadrature integration over the stock return realizations and cubic spline interpolation.

### 2.5 MODEL CALIBRATION

For the base case calibration, we follow Brown and Poterba (2000) and set the initial age of the husband to 65 years and that of the wife to 62 years, which they argue is representative of older US families. Our projection horizon is \( T = 39 \) years, i.e., the maximum age of the husband (wife) is 103 (100). We set the coefficient of relative risk aversion to \( \gamma = 5 \), the time preference rate to \( \beta = 0.96 \), and the bequest motive parameter to \( B = 2 \). These values are commonly used in the lifecycle portfolio choice literature (see e.g., Horneff et al., 2009). For the couple, we choose a consumption scaling factor of \( \phi_{s=1} = 1.3 \). This value is in line with the average scaling factor reported in Fernandez-Villaverde and Krueger (2007), who summarize the literature on household equivalence scales. This value is below the 1.7 used by the OECD (OECD, 1982), but higher than the empirically estimated 1.06 in Lazear and Michael (1980) and Nelson (1988).

Following Cocco, Gomes, and Maenhout (2005), the real riskless interest rate is set to \( 2\% \) \( (R_f = 1.02) \), and the parameters for the lognormal stock distribution are chosen, such that the risk premium is \( 4\% \) \( (E[R_t] - R_f = 4\%) \) and the stock volatility is \( 20\% \).

For the one-period survival probabilities \( (p_t^x, p_t^y) \) in the utility function (17), we use the US 2001 population life table from the National Vital Statistics Report 2005 (Arias, 2010). For pricing term life insurance contracts, we use composite mortality rates from the 2001 Commissioners Standard Ordinary (CSO) Mortality Table, which represents the mortality experiences of those covered by life insurance. Annuities are priced using...

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8 Throughout the article, we assume that all mortality rates remain constant. In particular, we do not account for mortality trends or cohort effects.

9 The CSO table was developed by the Society of Actuaries and the American Academy of Actuaries. In most states of USA, it serves as the legally required minimum standard for calculating the actuarial reserves of life insurance companies. The mortality rates reported...
the Annuity 2000 Mortality Table provided by the Society of Actuaries. The survival probabilities used for pricing insurance products differ from those of the general population. As a result, insurance premiums are not actuarially fair but include age-dependent implied loadings. With annuitants’ survival probabilities exceeding those of the general population at all ages, implied loadings vary between about 11.5% for joint annuities at the age of 65 years and about 31.5% for single annuities for males aged 85 years. These loadings serve insurance companies as safety margins, for example, to manage systematic longevity risk, and they also help to address the problem of adverse selection in the private annuity market.10 For most ages, survival probabilities from the CSO table also exceed those of the general population. Implied loadings for life insurance, therefore, range from about −18% to about 6%. Survival probabilities of life insurance purchasers exceeding those of the general population are also reported in Cawley and Philipson (1999), for the USA and in McCarthy and Mitchell (2010) for the USA and the UK. According to Cawley and Philipson (1999) this indicates that, in contrast to annuity providers, suppliers of life insurance are successful in distinguishing between high- and low-risk individuals through underwriting, and consequently limit coverage of high-risk individuals. In addition to the implied loadings discussed above, we assume explicit expense loadings of 2% on all insurance products, which is in line with self-reported costs by industry leaders, such as Vanguard.

Consistent with empirical evidence from the US annuities markets, the maximum age at which annuities can be bought is set to 85. While we are able to model annuities as illiquid long-term investments, we have to restrict ourselves to 1-year term life insurance contracts for computational reasons. Modeling long-term contracts would require two additional (continuous) state variables for tracking life insurance purchases in previous periods. Long-term contracts can, however, be qualitatively replicated by repeated purchases of 1-year term life insurance, provided that these are available over the complete horizon. Accordingly, we allow for life insurance purchases up to year T − 1, although this may allow excess flexibility in adjusting life insurance holdings. Offering short-term life insurance contracts at advanced ages could result in adverse selection. Individuals might try to benefit from private information regarding their health status and seek to

in the CSO table are higher than those observed by life insurance providers during the estimation period (1990–95) to account for estimation risk (see American Academy of Actuaries, 2002).

10 Finkelstein and Poterba (2004) provide strong empirical evidence for adverse selection in the UK annuity market. They show that the typical annuitant is longer lived than an average individual in the population.
purchase life insurance once their health deteriorates. Our discussion of the characteristics of the CSO life table shows that insurers are quite successful in screening the health status of potential customers for long-term life insurance contracts. If such screenings would be too costly for short-term contracts, insurance premiums would have to increase substantially, which might finally lead to a breakdown of the insurance market. In our model, however, individuals do not have private information regarding their survival probabilities as these are governed by the deterministic population life tables. Consequently, life insurance contracts can only be mispriced to the extent to which the CSO table differs from the population mortality table.

3. Results

3.1 Lifecycle Profiles—Base Case Calibration

This section analyzes the household’s optimal lifecycle behavior. To this end, we present average results from 10,000 simulated lifecycles based on the previously derived optimal controls. While these controls account for potential changes in the family state, we only present lifecycle profiles with both spouses being alive, i.e., we fix the family state to $s_t = 1$ in the simulation. This approach ensures that the displayed profiles focus on the decisions of couples and are not mixed with those of singles, whose annuitization behavior is already well documented in the literature.

In what follows, we assume that the couple retires at the age of 65/62 years\textsuperscript{11} with a total retirement wealth of 100 monetary units. With respect to the retirement nest-egg, we distinguish three initial allocations. First, the couple’s endowment consists only of liquid wealth, which can be (after expenditures for consumption and insurance products) invested in the bond and stock market. Second, the initial endowment consists of liquid wealth and pre-annuitized wealth in the form of a life-contingent retirement income that is reduced to 67% of the initial amount after the first spouse’s death. Such a K67% joint annuity can be considered as a portfolio of three annuities that pay the same benefits: a single-life annuity for both, the husband and the wife, as well as a joint annuity. Hence, retirement income is symmetrically distributed between the two spouses. US Social Security retirement benefits resemble such an annuity. Even if, for instance, only the husband has worked, the wife qualifies for marital benefits as long as

\textsuperscript{11} In what follows, we only refer to the husband’s age, keeping in mind that the wife is 3 years younger.
the husband is alive and for widow benefits after his death. The 3rd initial endowment is a combination of liquid wealth and asymmetrically distributed preexisting retirement income. Here, only half of the retirement income is drawn from a symmetric $K67\%$ joint annuity. The other half is provided by a single annuity for the husband. The retirement income will therefore be reduced to $83\%$ if the wife dies, but to only $33\%$ if the husband dies. This can be interpreted as a case, in which the husband receives a defined benefit pension without survivor benefits, on top of Social Security.\footnote{By default, US defined benefit plans provide a survivor benefit of $50\%$, unless both spouses decide otherwise. Here, we refrain from including survivor benefits, because otherwise this setting would not differ significantly from our second case with symmetrical annuitization.} In the second and the third allocation, preexisting retirement income levels are chosen such that the initial ratio of wealth to retirement income (wealth to income ratio) is 8, while still fixing the total wealth to 100 for comparability between the settings.\footnote{For the definition of the initial wealth to income ratio, we use the retirement income in the denominator and liquid wealth according to Equation (8) minus the retirement income in the numerator. This makes the model results comparable to the empirical data, since households receive income in monthly installments, rather than as annual income at the beginning of the year. The chosen value of eight is near the mean of the HRS sample reported in Section 4.2, Table III. While the prices of additional private annuities are calculated using annuitant mortality tables and include explicit costs, the present value of the preexisting retirement income is calculated based on actuarially fair annuity factors with population survival probabilities and without costs. This is justified because Social Security and employer-sponsored defined benefit plans face less risk of adverse selection and charge lower fees than providers of private annuities.} We first study these settings for couples without access to the private annuity market (Figures 1–3). Subsequently, we allow the couple with symmetrical (Figure 4) and asymmetrical (Figure 5) retirement income to buy additional annuities in the private market.

Figure 1 presents the expected lifecycle profile for a couple that is endowed only with liquid wealth and lacks access to the private annuity market. In the absence of any annuitized retirement income, the problem is effectively reduced to the Merton (1969) case. Accordingly, while liquid wealth (Panel A) is continuously depleted to finance consumption, the relative amount invested in stocks remains constant over time at $21.8\%$ (Panel D). Despite its bequest motive, the couple does not purchase any life insurance (Panel B). Since there is no alternative to holding liquid wealth for financing future consumption, they automatically hold sufficient assets to provide for bequests. Furthermore, the death of one spouse is not a negative shock, economically. The surviving spouse must consume less than the couple to draw the same utility, while financial wealth is unaffected by the death of the
spouse. Consequently, there is no risk for the couple that must be hedged through life insurance. Consumption is high early in retirement (Panel C), since they anticipate that one spouse may die and that subsequently only the survivor will need to finance consumption. As the couple ages, however, average consumption declines quickly. At the age of 95 years, the couple consumes less than half of what they consumed at the age of 65 years.

Figure 2 presents the case with symmetrically distributed preexisting retirement income. At the age of 65 years, about 64% of the total wealth is allocated to preexisting retirement income (in present value terms). The couple’s annual retirement income amounts to 4.02, which is reduced by 1.34 (33%) in case one of the partners dies (Panel C). In comparison to the previous case without any retirement income, liquid wealth and consumption decrease less rapidly, since the couple receives a constant lifelong income even at advanced ages. In general, consumption is higher than in the case with no retirement income, indicating that having symmetrically pre-annuitized wealth increases the couple’s welfare.

Between ages 65 and 80 years, there is small and diminishing demand for life insurance on the husband (Panel B). As the wife has higher survival probabilities than her husband, she is more likely to be widowed and would, in expectation, need to finance consumption for longer than the
widower. For this, she needs sufficient liquid wealth, since, on her husband’s death, retirement income falls by one-third, whereas consumption needs to decrease by about 23% only.\textsuperscript{14} Triggered by the provision motive, the husband holds some life insurance to help bridge the income gap that would emerge in case of his death. The initial life insurance face value, however, amounts to only 3.5 monetary units. This is less than both the household’s annual retirement income and consumption, such that life insurance payouts would contribute little to the widow’s consumption. The husband, on the other hand, is financially less affected by the early death of his wife. He faces the same disproportionate reduction in both retirement income and consumption needs, but he has a relatively ample supply of financial wealth due to his much shorter remaining planning horizon. Consequently, the couple will not purchase life insurance on the wife. A complete absence of any life insurance holdings at advanced ages confirms that life insurance is not purchased to finance bequests. In fact, with symmetrically distributed retirement income, bequest is only financed by liquid wealth.

The most pronounced difference between this and the first case is the much higher stock weight. Initially, 71.6\% of the financial wealth is

\textsuperscript{14} After the death of one spouse, the consumption scaling factor declines from 1.3 to 1 (\(=23\%\)).
invested in stocks, whereas the remaining 28.4% is allocated to bonds. With constant payouts for life, preexisting retirement income resembles the risk profile of a bond-like investment. Consequently, compared to Figure 1, the couple will hold more of its financial wealth in equities in order to adjust the household’s overall risk exposure. As age increases, the ratio of financial wealth to this annuitized wealth increases. While the present value of the bond-like retirement income quickly depreciates, the couple retains some financial wealth to finance bequest. Consequently, the couple reduces the fraction of financial wealth invested in stocks over time. The share of total wealth (i.e., the sum of stocks, bonds, and the present value of preexisting annuity claims) invested in stocks, however, is projected to remain fairly constant over time, between 22% and 24%, which is nearly the same value as in the first case presented in Figure 1.

Figure 3 presents expected lifecycle profiles when preexisting retirement income is asymmetrically distributed between the partners. The development of liquid wealth and the present value of preexisting retirement income are similar to the symmetrical case. Total household income is slightly higher than in the previous case and is tilted much more toward annuities for the husband. At the same time, the couple must cut consumption in order to

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15 See also Chai et al. (2011) and Horneff, Maurer, and Stamos (2008b) for this point.

16 Due to the asymmetrical distribution of retirement income, the couple’s preexisting annuity portfolio holds more of the cheaper single annuities for the husband. As we
finance substantial life insurance purchases for the husband. If the husband
dies first, the widow’s income drops to only 33% of its previous level, well
short of what is required to finance a utility-equivalent consumption stream.
Consequently, there is strong incentive to provide for her consumption with
additional liquid wealth from life insurance payouts. In contrast, when the
wife dies first, the widower receives 83% of the couple’s previous income, so
retirement income decreases less than consumption requirements. As she
does not need to provide for the husband, there is no demand for life
insurance by the wife. At the age of 65 years, the couple spends 0.46 on
purchasing life insurance for the husband with a face value of 27.2,
approximately 70% of the financial wealth. This is sufficient to finance the
widow’s consumption for over 6 years in expectation. With increasing age, the
wife’s expected remaining lifetime decreases and so does the need to hold life
insurance on the husband. Consequently, the face value declines over time.

The fraction of financial wealth invested in stocks is comparable to the
symmetrical case, even though the decline at advanced ages is less
pronounced. The fraction of total wealth held in equities is again slightly
above 20% and remains almost constant over time.

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again fix total initial wealth at 100 and the wealth to income ratio at 8, this implies that
total household retirement income is slightly higher than in the case with symmetrical pre-
annuitization, 4.28 instead of 4.02.

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**Figure 4.** Expected life cycle profiles with symmetrically distributed preexisting retirement
income and with access to the private annuity market. Retirement income is a K67% joint
annuity, such that liquid wealth to income ratio is 8. Panel (A): liquid wealth and present
value of (preexisting + private) annuity claims. Panel (B): face values of life insurance
holdings for husband and wife. Panel (C): annual payments from preexisting and additional
private annuities for husband, wife, and joint as well as the couple’s consumption. Panel
(D): fraction of financial wealth FW (stocks and bonds + present value of private annuities)
and total wealth TW (FW + present value of preexisting annuity claims) invested in stocks.
*Notes:* see Figure 1.
Next, we allow the couple to also purchase additional annuities in the private market. Figure 4 presents the results for the case with symmetrically distributed preexisting retirement income. In the first 20 years after retirement, the patterns for wealth, life insurance holdings, income, consumption, and asset allocation (Panel A–D) are essentially identical to the case without access to the private annuity market (Figure 2), since the couple has effectively no demand for private annuities. Despite the fact that the implicit loading factors of the annuities increase with age, the couple postpones purchasing these annuities in an effort to hedge against the risk of early death of one of the spouses. Instead of buying expensive joint annuities early in retirement, they prefer to wait. In case one of the spouses dies, the surviving spouse can buy the cheaper single annuities for him or herself. Only at the age of 85 years, the last year in which annuities can be purchased in the private market, does the couple substantially increase its annuity holdings. While the wife has a higher remaining lifetime in expectation, even at advanced ages, it is not unlikely that a husband will actually outlive his wife. Consequently, the couple refrains from considerably increasing the individual annuity income of either of the spouses and instead purchases the more expensive joint annuities, which also provide income to the last survivor. As it is more likely that the wife outlives her husband, the couple slightly increases her single annuity income.

Interestingly, access to the private annuity market has hardly any influence on optimal life insurance holdings (Panel B). Compared to the situation with symmetrical preexisting retirement income and no access to the private annuity market (reported in Figure 2), this case only differs in that the couple will purchase negligible amounts of life insurance for the wife after the age of 85 years. These purchases can be attributed to the small amount of additional single annuities for the wife that the couple buys at the age of 85 years. The substantial purchases of joint annuities at the age of 85 years, on the other hand, do not trigger life insurance purchases, despite the reduction in liquid wealth available for possible bequests (Panel A). This is further evidence for the hypothesis that term life insurance is not held to provide for bequests.

Next, we again turn to the allocation of financial wealth to equities. In the presence of private annuities, we define financial wealth as the sum of stocks, bonds, and the present value of private annuities, as the couple has deliberately invested in all of these assets. Up to the age of 85 years, the fraction of financial wealth invested in stocks is the same as in the case

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17 This definition of financial wealth is common in the literature on lifecycle portfolio choice with annuity markets (see e.g., Horneff et al., 2008a, 2010). It is also in line with
without access to the private annuity market. The reason is that up to this age, the couple basically buys no additional private annuities. Due to the last-minute annuity purchases at the age of 85 years, the equity exposure drops by about 5%. The purchase of additional annuities is mainly financed by selling bonds, due to their comparable risk characteristics with respect to financing consumption. Yet, only using bonds to purchase annuities would disproportionately increase the equity exposure of the remaining liquid assets, putting the potential bequest at risk. Hence, the couple partially finances annuities by selling equities, which results in the slight reduction of the fraction of financial wealth invested in stocks.\textsuperscript{18} This reduction is less pronounced for stock holdings relative to total wealth (i.e., the sum of financial wealth and present value of preexisting retirement income), which again remains fairly constant at around 20%.

Figure 5 finally presents results for a couple with asymmetrically distributed preexisting retirement income, which has access to the private annuity market. Retirement income consists of a single annuity for the husband and a \( K67\% \) joint annuity, such that liquid wealth to income ratio is 8. Panel (A): liquid wealth and present value of (preexisting + private) annuity claims. Panel (B): face values of life insurance holdings for husband and wife. Panel (C): annual payments from preexisting and additional private annuities for husband, wife, and joint as well as the couple's consumption. Panel (D): fraction of financial wealth FW (stocks and bonds + present value of additional private annuities) and total wealth TW (FW + present value of preexisting annuity claims) invested in stocks. \textit{Notes}: see Figure 1.

\textsuperscript{18} The fraction of liquid wealth invested in stocks, on the other hand, increases after the purchase of additional private annuities.
household will immediately begin purchasing single annuities for the wife to reduce the degree of asymmetry in the distribution of lifelong income. From the age of 65 to 85 years, the wife’s annuity income is built up only gradually, as there is some risk that she might die early. At the age of 85 years, the couple makes a final bulk purchase of single annuities for her and also buys a small amount of additional joint annuities. Having a bequest motive, however, the couple cannot fully deplete liquid wealth and is therefore limited with respect to last-minute annuity purchases. As a result, lifelong annuity income is still asymmetrically distributed after the age of 85 years. Even with access to the private annuity market, a couple with asymmetrically distributed preexisting retirement income will hold large amounts of life insurance on the husband. Face values are slightly lower than in the case without access to additional annuities, as the wife now has the opportunity to use life insurance payouts more efficiently, by buying additional annuities for herself, insuring against longevity risk, and cashing in the survival credit. After the age of 85 years, even the wife holds some life insurance. This and the purchase of additional joint annuities indicate that even the better-endowed widower cannot finance future consumption and bequest motive with his income, as liquid wealth has already been reduced substantially. In this case with comparably little liquid wealth remaining, we therefore cannot rule out that life insurance is purchased to finance a bequest. Yet, the face values of the wife’s life insurance are still much lower than the remaining liquid savings.

As in the case discussed before, the fraction of financial wealth invested in stocks decreases over time because the couple continuously purchases additional annuities from the private market. Now, the drop in the stock fraction at the age of 85 years is even more pronounced, since the value of last-minute annuity purchases exceeds the amount of funds held in bonds. Therefore, the couple must finance the purchase of additional annuities through disproportionately selling stocks. For the stock fraction of total wealth, the drop is less distinctive.

As we focus on the optimal decisions of couples, we have not yet reported model predictions for a surviving spouse. In the following paragraphs, we briefly describe optimal expected lifecycle behaviors for a widow, whose husband dies at the age of 70 years, i.e., we condition the family state to $s = 3$ from the age of 70 years on.19

In all five scenarios discussed above, the widow is endowed with ample liquid wealth relative to the (reduced) income. When having access to private

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19 Full lifecycle profiles for this and alternative ages of death are available from the authors on request.
annuities, the widow immediately purchases a substantial amount of annuities. These purchases result in the same wealth to income ratio, independent of whether preexisting retirement income was symmetrically or asymmetrically distributed.

In all the five cases, the widow optimally holds liquid wealth to finance bequests. Accordingly, life insurance holdings are zero throughout the widow’s remaining lifetime for most cases. Only with access to the private annuity market, the widow holds negligible amounts of life insurance at higher ages, since liquid savings were considerably decreased by the annuity purchases.

For the one case without any annuity income, the stock weight chosen by the widow is identical to the one of the couple (Figure 1). In the other four cases, the widow reacts to the loss of the husband’s income by reducing the fraction of financial wealth invested in stocks. When lacking access to additional private annuities, she increases her bond holdings, while she otherwise increases the share of financial wealth invested in annuities.

To sum up, subsequent to the adjustments triggered by the husband’s death, annuitization and portfolio choice policies resemble those previously reported for single-agent models (e.g., Horneff, Maurer, and Stamos, 2008a; Pang and Warshawsky, 2010). The lack of life insurance demand when holding measurable liquid wealth is in line with Inkmann, Lopes, and Michaelides (2011). In contrast to their results, however, it is optimal to keep some liquid wealth instead of depleting it completely and subsequently buying life insurance to finance bequest.

We conclude this section with a few remarks on optimal equity exposures reported above. Based on US household data from the 2001 Survey of Consumer Finances, Gomes and Michaelides (2005) find that, subject to participating in the equity markets, stock fractions of households in retirement decrease slightly with age, from an average of 55% for those aged 65–74 years to around 50% for those aged 75 or more. In line with this, when including preexisting retirement income, our model predicts equity exposures that gradually decrease as the household ages. Depending on the scenario, our model predicts average stock fractions ranging from 64% to 71% for those aged 65–74 years and from 46% to 65% for those aged 75 years or more, slightly above empirical findings.20

Gomes and Michaelides (2005) also report that only about 40% of the elderly participate in the stock market. This finding, however, cannot be

20 Further results, available on request, suggest that average stock fractions decline by about 15%, if funds, which are earmarked to finance next period’s consumption, must be held in bonds for transaction purposes.
endogenously reproduced by many models including ours. Explanations usually offered for this deficiency of our and related models include neglecting transaction costs, health shocks, related medical expenses, liquidity needs for transaction purposes, and behavioral aspects. We address this issue by repeating our analysis for a couple, who is exogenously prevented from participating in the stock market. Together with the implications of variations in other important parameters for the demand for life insurance and private annuities, we present our findings for nonstockholders in the next section.

3.2 SENSITIVITY AND WELFARE ANALYSIS

In this section, we analyze the sensitivity of life insurance demand to key model parameter values. To this end, we conduct comparative statics analyses beginning with our baseline calibration and varying one parameter at the time. The parameters of interest include the initial wealth to income ratio, the strength of the (pure) bequest motive, the level of jointness in consumption, the age difference between husband and wife, and the couple’s relative risk aversion. Moreover, we study the effect of restricting access to the stock market, the impact of “broken heart effects” (i.e., interdependence of mortality rates), and the implications of no implied loadings in life insurance premiums. Subsequently, we analyze the welfare implications of having access to term life insurance for our baseline calibration as well as the alternative parameterizations. For both sensitivity and welfare analysis, we concentrate on the two scenarios with asymmetrically distributed preexisting retirement income presented in Figures 3 and 5, as the other scenarios were shown to just generate negligible life insurance demand. Results are summarized in Table I.

The first column of Table I presents the effects of parameter variations on life insurance demand for a couple lacking access to the private annuity market. As the age-related decrease in face values proves to be comparable for the parameterizations under scrutiny, we take a parsimonious approach and study the differences in life insurance demands by comparing the face values of the husband’s life insurance at the age of 65 years. Here, we present the face values in multiples of total household preexisting retirement income, in order to get comparable numbers for alternative wealth and income levels. In the base case, our base case is again parameterized as follows: Initial wealth to income ratio of 8, access to the stock market, an age difference between the spouses of 3 years, relative risk

21 Our base case is again parameterized as follows: Initial wealth to income ratio of 8, access to the stock market, an age difference between the spouses of 3 years, relative risk
relative to household income, there is substantially less demand for life insurance. For instance, an initial wealth to income ratio of 20 would result in a first-year life insurance face value of 5.19 times annual income. In this situation, losing the husband’s income is less of a financial shock as the wife has ample liquid resources to finance future consumption. In contrast, with little liquid wealth relative to income, face values are only slightly higher than in the base case: 6.39 times annual income for a wealth to income ratio of 2. While there is strong need to insure against a possible income loss, life insurance premiums quickly eat up a measurable share of the little liquid resources available. Consequently, it is not optimal to substantially increase life insurance holdings.

Inkmann and Michaelides (2012) find that there is no participation in the life insurance market if the intentional bequest motive is eliminated. Yet, this result is based on a portfolio choice model with only a single representative agent; therefore, that work cannot differentiate between a pure bequest motive and a provision motive. In contrast, we find that a couple lacking a (pure) bequest motive has virtually the same demand for life insurance as our baseline household; 6.34 times annual income. In other words, our results show that for a couple with baseline wealth and asymmetrically distributed retirement income, life insurance demand is almost exclusively driven by the provision motive.

In contrast, the consumption scaling factor has a large impact on life insurance demand. When the factor is smaller, a widow must consume almost as much as the couple did before her husband died, in order to maintain her desired level of utility. Since after the husband’s death, she will receive a much smaller income, the couple must hold a substantial amount of life insurance beforehand to provide sufficient additional liquid wealth later. For a consumption scaling factor of 1.1, the face value amounts to 8.2 times annual income; a higher scaling factor will, in turn, reduce life insurance holdings (to 4.8 for a factor of 1.5).

Couples with a larger age difference between the two partners will also have a higher demand for term life insurance: 6.58 times annual income, if the wife is 8 years younger than her husband. Being much younger, the wife has a high probability of outliving her partner. At the same time, she will have to live on this reduced income for a longer period. The resulting need for supplemental financial wealth drives up life insurance purchases.

Interestingly, both low- and high-risk aversion result in slightly lower demand for life insurance. A couple with a parameter of relative risk aversion of $\gamma = 5$, time preference of $\beta = 0.96$, consumption scaling factor of $\phi = 1.3$, and bequest motive strength of $B = 2$. 
aversion of \( \gamma = 2 \) \((\gamma = 8)\) will purchase life insurance with an initial face value of 6.05 (6.21) times their annual income. For a lower risk aversion parameter, the couple’s expected utility is less affected by the risk that the widow might face a substantial drop in income and consumption, so they demand less life insurance. A very risk averse couple, on the other hand, consumes less early in retirement and retains more liquid wealth. Both effects reduce the potential widow’s exposure to the inevitable income loss when the husband dies and, hence, the demand for life insurance on the husband decreases.

Without access to the stock market, the couple cannot cash in on the equity premium. Consequently, it is less wealthy in expectation, and the wife faces a higher risk of suffering a substantial drop in consumption. To compensate for this possible reduction in wealth, the couple increases initial life insurance demand to 6.44 times annual income.

Early in retirement, mortality rates of life insurance holders are below that of the average population, as discussed above. This results in negative implied loadings when assuming that individuals die according to the population mortality table, which in turn drives up life insurance demand. To quantify this effect, we next assume that the couple’s survival probabilities used to discount future expected utility are the same as those used for pricing the life insurance, i.e., the CSO table. In this scenario, life insurance demand drops by about 10% to 5.8, as higher survival rates render life insurance less attractive.

Finally, we study the impact of interdependence of mortality rates on life insurance demand. Several studies provide empirical evidence that the death of one partner may temporarily increase the mortality probability of the surviving spouse (see, for instance, Young, Benjamin, and Wallis, 1963; Parkes, Benjamin, and Fitzgerald, 1969; Helsing and Szklo, 1981; Jagger and Sutton, 1991). This so-called “broken heart effect” has been shown to be more pronounced for widowers than for widows. In what follows, we study the impact of a very strong broken heart effect, assuming that the mortality of a widower (widow) will not only be temporarily but permanently increased by 35% (15%).\(^{22}\) These numbers are at the higher end of what has been found empirically. Despite this substantial broken heart effect, the initial life insurance face value still averages 6.19 times annual income, only marginally lower than in the base case. This result is driven by two offsetting effects. Due to the rise in the widow’s mortality,

\(^{22}\) To the best of our knowledge, providers of neither annuities nor life insurance contracts account for broken heart effects in pricing their products. Consequently, we also refrain from doing so and only adjust the survival probabilities in the value function.
consumption must be financed for a shorter time period, driving down life insurance demand. At the same time, the broken heart effect generally reduces the rate of time preference and hence renders the couple more impatient. Consequently, the couple’s consumption is higher early in retirement, as is the wife’s utility-equivalent consumption need in widowhood. With lower liquid wealth remaining due to higher consumption, the demand for life insurance increases.

Turning to the scenario with access to additional annuities, Column 2 of Table I shows that, for all calibrations except low risk aversion, life insurance demand is lower than in the case without access to private annuities. Purchasing annuities for the wife reduces income asymmetry, and hence life insurance demand related to the provision motive. At the same time, a widow with access to private annuities needs less additional liquid wealth from life insurance payouts for financing consumption. At older ages, annuities outperform bonds and even stocks due to the increasing survival credit. Accordingly, life insurance payouts can be invested with

### Table I. Effect of parameter variations on life insurance holdings

<p>| Effect of parameter and setting variations on life insurance demand and utility gains from access to life insurance for both cases with and without access to the annuity market. Life insurance demand: face value of husband’s life insurance in first period in multiples of total household income for asymmetrical pre-annuitization. Utility gained from life insurance: relative increase in the couples’ certainty equivalent of indirect utility at the age of 65 years gained by the access to the life insurance markets, all other parameters being equal. Base case parameterization: ( \gamma = 5, \beta = 0.96, \phi = 1.3, B = 2 ) a wealth to income ratio of 8, and access to the stock market. Source: authors’ calculations. |
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<th>Effect of parameter and setting variations on life insurance demand and utility gains from access to life insurance for both cases with and without access to the annuity market. Life insurance demand: face value of husband’s life insurance in first period in multiples of total household income for asymmetrical pre-annuitization. Utility gained from life insurance: relative increase in the couples’ certainty equivalent of indirect utility at the age of 65 years gained by the access to the life insurance markets, all other parameters being equal. Base case parameterization: ( \gamma = 5, \beta = 0.96, \phi = 1.3, B = 2 ) a wealth to income ratio of 8, and access to the stock market. Source: authors’ calculations.</th>
<th>Life insurance demand</th>
<th>Utility gained from life insurance</th>
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<tr>
<td>Without annuities</td>
<td>With annuities</td>
<td>Without annuities (%)</td>
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<td>Base case</td>
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<td>Wealth/income = 2</td>
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<td>Cons. scaling ( \phi = 1.5 )</td>
<td>4.80</td>
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<td>Age difference: 8 years</td>
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<td>Risk aversion ( \gamma = 8 )</td>
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<td>No stock market access</td>
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<td>Higher survival prob.</td>
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<tr>
<td>Broken heart effect</td>
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<td>5.61</td>
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higher return than when the widow is prevented from participating in the private annuity market.

Next, we seek to quantify the welfare gains of having access to term life insurance. To this end, we determine the changes in certainty equivalent wealth and annuity income at the age of 65 years for the different parameterizations studied above. The certainty equivalent is defined as the inverse of the value function:

\[ CE_t(W_t, A_t, s_t) = \left( (1 - \gamma) \cdot J_t(W_t, A_t, s_t) \right)^{1/\gamma} \]

For a given family state \( s_t \) and fixed relative annuity payments \( a_t (= A_t/\bar{W}_t) \), the certainty equivalent is proportional to wealth \( \bar{W}_t \). Consequently, an \( \alpha \)% increase in the certainty equivalent corresponds to an \( \alpha \)% increase in both liquid wealth and annuity income.

The findings appear in Columns 3 and 4 of Table I. Lacking access to the private annuity market, the baseline couple enjoys welfare gains of 5.4% if able to purchase term life insurance. Comparable utility increases are found for the cases with no bequest motive, a large age difference between the partners, no access to the stock market, a strong broken heart effect, and no implied loadings for life insurance premiums. The differences in initial face values are comparably small when changing the relative risk aversion and the wealth to income ratio, but the differences in utility gains are much more pronounced. In particular, we find substantially higher increases in utility due to access to life insurance for couples with a low wealth to income ratio (11.5%) and high-risk aversion (7.6%). At the same time, couples with a high wealth to income ratio and low-risk aversion only have modest utility increases (of 1.7% and 2.2%, respectively). Couples with high consumption scaling also benefit less from having access to life insurance (3.5%). For those with low consumption scaling, on the other hand, term life insurance again proves to be particularly valuable (8.2%).

As discussed above, access to the private annuity market reduces the appeal of term life insurance, since a couple can offset an income asymmetry by purchasing single annuities for the wife. Yet, access to life insurance still substantially increases welfare for most parameterizations. With access to private annuities, welfare gains are in general about 0.5% lower without them. If a couple has a low wealth to income ratio, or no bequest motive, welfare gains are in fact considerably higher, by 0.8% and 1.8%, respectively. Here, it is highly desirable for a surviving spouse to purchase a substantial amount of additional private annuities. Term life insurance can
offer the required financial means for those purchases and, hence, is particularly valuable.

3.3 PURE BEQUEST MOTIVE AND LIFE INSURANCE DEMAND

Thus far, we have showed that term life insurance will be purchased mainly to overcome asymmetries in the distribution of retirement income, and that pure bequest motives have little effect on life insurance holdings. In what follows, we will show that a key driver of this result is the couple's endowment of liquid wealth. To this end, Table II presents the couple's life insurance demand for alternative wealth to income ratios. Here, we assume that preexisting retirement income is symmetrically distributed between the spouses, in order to reduce the impact of the provision motive. Both spouses hold life insurance in this setup, so we report the couple's combined initial life insurance face value, again expressed in multiples of annual income.

The baseline couple with bequest motives and a wealth to income ratio of 8 will hold life insurance with a face value of 0.87 times annual income, which is only about one-seventh of the 6.35; we reported for the corresponding case with asymmetrically distributed preexisting retirement income (Table I). Lacking a bequest motive, an otherwise equal couple demands 0.79. Consequently, only about 9% of the life insurance holdings can be attributed to a bequest motive. In contrast, if a couple has no liquid wealth at all, including a bequest motive increases life insurance demand from 1.95 to 2.79 times annual income. In other words, about 30% of the life insurance demand by the wealth-poor is directly related to the pure bequest motive.

This wealth effect becomes even more apparent when we increase the consumption scaling factor from $\phi = 1.3$ to $\phi = 1.5$. Under this parameterization, both retirement income as well as utility-equivalent consumption will drop to two-thirds, when one spouse dies. Consequently, life insurance demand can mainly be attributed to a bequest motive. Here, we find that there is no life insurance demand for our baseline wealth to income ratio of 8. That is, this couple is sufficiently endowed with liquid wealth to finance a bequest. In contrast, with zero liquid wealth, the couple holds life insurance worth 1.4 times its annual income. Face values quickly decrease with increasing wealth levels: for a wealth to income ratio of 2, hardly any bequest-related life insurance demand remains.23

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23 In further analyses, not reported here, we stipulate that the husband dies immediately before retirement, leaving the surviving spouse without a provision motive but with a pure...
This wealth effect for a two person couple is similar to that reported by Inkmann, Lopez, and Michaelides (2011) for a lifecycle model with a (single) representative agent. It can be explained as follows: with little or no liquid wealth, quickly accumulating sufficient funds to finance the relatively strong pure bequest motive—the targeted bequest amounts to about two times the annual consumption—results in a notable consumption discontinuity early in retirement. This can be circumvented by purchasing initially cheap term life insurance, which provides high payments in case both partners die. For households with sufficient liquid wealth to finance, the desired level of intergenerational wealth transfer, however, the (unconditional) risk-return profile of life insurance policies (i.e., low expected return and high-return volatility) is less appealing than that of a balanced bond/stock portfolio.

**4. Empirical Analysis of Life Insurance Demand**

**4.1 MOTIVATION**

The central insight from our normative model-based analysis is that asymmetries in spousal retirement income streams are the key determinant of an older couple’s demand for private annuities as well as term life insurance. Decisions to participate in insurance markets are mainly driven by a provision motive, i.e., the desire to ensure that the death of one spouse bequest motive. For wealth to income ratios of 0, 2, and 8, her life insurance demands average 1.61, 0.16, and 0 times annual income, respectively. This supports our findings for the two-person household.
will not be excessively harmful to the consumption opportunities of the surviving spouse. Pure bequest motives, on the other hand, are shown to play only a minor role in explaining a couple’s demand for term life insurance. Only those with very little liquid wealth are predicted to boost their life insurance holdings due to bequest motives.

This raises the question as to whether our model predictions are in line with empirically observed behavior of retired couples. To address this issue, we conduct an econometric analysis aiming at identifying the drivers of life insurance demand in late life based on survey data for elderly US couples. We focus on life insurance and disregard private annuities, as previous studies have already documented that actual demand for private annuities is negligible.24 Our analysis shows that, in accordance with our model predictions, asymmetry in the distribution of pension income between the spouses is a central explanatory variable of both the probability of holding life insurance as well as the amount held. On the other hand, proxies for a pure bequest motive, in particular the number of children, have no significant impact on life insurance holdings.

In what follows, we first describe the construction of our data set and provide descriptive statistics on the main covariates. Subsequently, we will present the empirical results of our regression analyses of the demand for life insurance.

4.2 DATA SET AND DESCRIPTIVE STATISTICS

The empirical analysis draws on the HRS, a biannual longitudinal panel study of American households over the age of 50 years.25 Specifically, we use the RAND HRS Data file provided by RAND Center for the Study of Aging and analyze survey data from HRS waves 5–10, covering the years 2000–10.

The group we explore includes only couple households with exactly two respondents of different sex, excluding all “divorced or separated households.” Of the remaining couples, 99.4% are married and 0.6% are partnered. To study households similar to those in our model, we exclude households in which the husband is less than 65 years old. Moreover, we only include couples that receive Social Security retirement income for at

24 For example, Johnson, Burman, and Kobes (2004) report that only 6% of the US adults aged 65 years and older receive income from (own and spouse’s) private annuities. Moreover, they show that, on average, private annuity income (wealth) only accounts for a mere 1% of the total household income (wealth).

25 For more information on the HRS, we refer to the survey website at the University of Michigan: http://hrsonline.isr.umich.edu/index.php.
least one of the spouses and exclude those that receive any labor income. To avoid distortions due to outliers, we exclude those households that are in the top 5% of both the financial wealth and the household income distribution, as well as those with negative financial wealth. This leaves us with 2,313 households.

Several key variables required for the analysis must be constructed: household income is calculated as the sum of Social Security retirement benefits and pension annuities of both spouses. Financial wealth is derived as the sum of saving and checking accounts, certificates of deposits, government saving bonds, treasury bills, stocks, mutual funds, and the value of individual retirement accounts (IRA) and Keogh accounts minus nonmortgage debts. Hence, financial wealth excludes housing wealth and the present value of Social Security benefits or pension annuities. Moreover, the survey does not directly distinguish between holding term life insurance and endowment plans. We identify term life insurance holders as holders of life insurance that does not build up any cash value.

We must also construct a measure of asymmetry in the distribution of retirement income between the spouses. Here, we concentrate on private pensions, typically originating from occupational defined benefit plans, and we measure asymmetry by the pension income share, i.e., the ratio of a spouse’s private pension income and the couple’s total private pension income. Income asymmetry reaches its maximum in case one spouse receives all pension income, whereas the other receives none. In this case, the sole receiver of private pension income has a pension income share of one, whereas that of his or her partner is zero.

When pension income is symmetrically distributed, each spouse’s pension income share is 0.5. We disregard Social Security when measuring income asymmetry, as it represents an annuity with effectively symmetrical income distribution. Due to generous survivor provisions, a widow receives the same Social Security income as a widower.

Table III provides sample means of selected couple household variables, aggregated over the age groups 65–69, 70–74, and 75–79. About one-third of the sampled husbands and wives hold term life insurance, with participation of husbands exceeding that of wives by 3–6%. These rates are almost stable over all considered age groups. Life insurance face values, on the other hand, exhibit considerable variation as individual’s age. Conditional on participating in the term life insurance market, face

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26 A table with more elaborate descriptive statistics is provided in the Appendix Table AI.
values average around $68,000 for husbands aged 65–69 years. Those in
the age bracket 75–79 only hold life insurance with an average face value of
around $31,000. Face values of life insurance held by wives also decline
sharply with age. At the same time, they fall substantially short of those
held by husbands. For wives aged 65–69 years, face values average $45,457,
only two-thirds of the amount held by husbands. In their late 70s, wives’ face
values only come to around 60% of that of their husbands.

Turning to private pensions, our data set exhibits even greater
asymmetries in participation and income between husbands and wives.
About 60% of the husbands receive income from private pensions, 58% in
the age group 65–69 years, and 62% of the husbands in their late 70s. For
wives, these numbers halve. Only 28% (33%) of the wives aged 65–69
(75–79) years receive pension income other than Social Security.
Conditional on either of the spouses receiving pension income, the
husbands’ share of this income averages 77% in the late 60s and 75% in
the late 70s.

4.3 ECONOMETRIC EVIDENCE

We identify the key drivers of couples’ demand for term life insurance in two
steps. First, we conduct a probit analysis with respect to the propensity to
hold insurance. Second, we estimate an OLS regression on life insurance face
values for insurance holders.

Table IV presents marginal effects on a spouse’s probability of holding life
insurance. These are calculated for a baseline nonstockholding 65-year-old
male spouse, who has a wife 3 years younger (sample mean: 2.55), three
children (sample mean: 3.2), household log financial wealth of 10.97, and
log household income of 10.24 (both numbers equal to sample means). The
pension income share is set to 50%, i.e., pension income in the base case is
symmetrically distributed between the two spouses. In accordance with our
model predictions in Figures 2–5 and Table I, the likelihood of holding life
insurance is lower for wives than for husbands. Moreover, it decreases with
increasing age and financial wealth, while it increases with household income
and, especially, with the asymmetry in the distribution of private pension
income. We find that a wife’s probability of holding life insurance is 2.63%
below that of a husband. Increasing a spouse’s age by 10 years or the
household’s log financial wealth by one standard deviation (i.e., 2.21) has
a similar impact on the probability of holding life insurance (i.e., reduced by
2.61% and 2.69%, respectively). We also find that the decision to (actively)

27 All dollar amounts are adjusted for inflation and denoted in 2010 dollars.
participate in the stock market reduces life insurance participation by 3.10%. In turn, if the spouse is the sole earner of pension income, his or her participation probability increases by 4.2%, and a one standard deviation (0.55) increase in log household income results in a 3.52% higher probability of holding life insurance. Moreover, Table IV suggests that the number of children, which may be a proxy for having a pure bequest motive, has only a negligible and insignificant impact on the likelihood of participating in the life insurance market. Increasing the number of children by one boosts the probability by only 0.43%. Finally, the difference in age between spouses also has an insignificant impact on the decision to hold life insurance.

The results of our OLS regression on life insurance face values are reported in Table V. Again, household income and asymmetry in the distribution of pension income are found to have statistically highly significant positive impacts on individual life insurance holdings, whereas female and age exhibit highly significant negative coefficients. As in our previous analysis, the number of children has no significant impact on life

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**Table III.** Sample averages of key quantities in the HRS

Averages of key household quantities for different age groups (age of husband) of the HRS waves 5–10. The sample consists of married or partnered households in which the husband is at least 65 years old, at least one spouse receives social security retirement, and none receive any labor income. To avoid outliers households with negative financial wealth as well as the top 5% both in financial wealth and household income distribution are excluded. Term life insurance participation equals one, if life insurance holdings but no endowment plans are reported, zero otherwise. The average life insurance face values (in 2010 dollars) are conditioned on term life insurance holdings. Pension participation equals one, if positive pension income is reported, zero otherwise. Husband pension income share is defined as pension income of the husband divided by the sum of the pension income of both spouses and conditioned on at least one spouse receiving pension income. Source: authors’ calculations.

<table>
<thead>
<tr>
<th>Age group</th>
<th>65–69</th>
<th>70–74</th>
<th>75–79</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wealth/income ratio</td>
<td>8.26</td>
<td>7.59</td>
<td>7.07</td>
</tr>
<tr>
<td>Term life insurance participation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Husband</td>
<td>0.36</td>
<td>0.35</td>
<td>0.34</td>
</tr>
<tr>
<td>Wife</td>
<td>0.33</td>
<td>0.29</td>
<td>0.31</td>
</tr>
<tr>
<td>Term life insurance face values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Husband</td>
<td>67,910</td>
<td>47,813</td>
<td>30,807</td>
</tr>
<tr>
<td>Wife</td>
<td>45,457</td>
<td>28,191</td>
<td>18,816</td>
</tr>
<tr>
<td>Private pension participation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Husband</td>
<td>0.58</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>Wife</td>
<td>0.28</td>
<td>0.31</td>
<td>0.33</td>
</tr>
<tr>
<td>Pension income share of husband</td>
<td>0.77</td>
<td>0.77</td>
<td>0.75</td>
</tr>
</tbody>
</table>
insurance holdings. In contrast to the results from our probit analysis, where financial wealth reduced the life insurance participation probability, wealth now has a positive and significant impact on life insurance face values for those who participate. Moreover, the age difference now has a small but statistically significant positive impact, whereas stock market participation is not associated with life insurance faces values. In order to gain more insight into the economic significance of our regression results, Table V also presents the marginal effects of variations in the regressors on life insurance face values, derived by calculating the predicted face value for the baseline individual from our probit analysis and then relating this value to the predicted face value that results from a deviation in the respective regressor. The face value of life insurance held by a female spouse is predicted to be about 47% below that of a male spouse. An increase in age of 10 years will also almost halve the predicted face value (−48.4%). Being the sole receiver of the pension income increases the face value of this spouse's life insurance by 9%. A one standard deviation increase in log financial wealth (log household income) drives up face values by 15.2% (9.8%). Compared to these numbers, changes in stock market participation, age differences, and the number of children have economically insignificant impacts on life insurance face values.

To sum up, our econometric analysis of the key factors driving the demand for term life insurance of retired couples reveals a statistically significant positive relationship between the asymmetry of the couple’s pension income and the probability of holding life insurance policies, as well as the corresponding face values. In contrast, other variables that can be seen as covariates for a pure bequest motive, such as the number of children, have no significant impact. These results mesh well with the predictions from our theoretical lifecycle model and support the hypothesis that term life insurance demand of retired couples is mostly driven by a provision motive, and not by a pure bequest motive.28

28 In further analyses, not reported here, we repeat our empirical study for couples with symmetrically distributed pension income in order to test our theoretical findings from Section 3.3. When comparing poor and rich couples, the descriptive statistics show no significant differences in life insurance participation rates or life insurance face values relative to household income. Moreover, the central findings of our regression analyses of the overall sample are also confirmed. In particular, the impact of the number of children on life insurance holdings is still insignificant for both rich and poor couples. Our theoretical findings in Section 3.3 suggested a strong increase in life insurance holdings of poor couples in the presence of a bequest motive. Consequently, the lack of increased life insurance holdings in the data could indicate the absence of a strong bequest motive among poor couples.
5. The Annuity Puzzle Revisited

Our findings in Section 3 suggest that the representative couple will exhibit substantial demand for private annuities, particularly if preexisting pension income is asymmetrically distributed between the two spouses. In an effort to reconcile our model predictions with the negligible demand for private annuities observed empirically (see e.g., Johnson, Burman, and Kobes, 2004), we subsequently conduct a heterogeneity analysis of optimal annuity demand for alternative endowments of financial wealth. For the sake of comparability between settings, we abstract from timing decisions and reformulate our optimization problem, such that the couple is allowed to purchase private annuities only once at the beginning of retirement.

We group the couples from our HRS data set into nine bins, representing the 1st–9th wealth decile. Specifically, bin \( n \) includes all couples whose financial wealth falls into the interval between the

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29 As we only analyze one-time annuity purchases at the beginning of retirement, we restrict our data sample to couples in the relevant age group, i.e., between 65 and 69 years. Consequently, this matches the sample of the first age bin in Table III.

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Table IV. Probit regression for term life insurance participation in the HRS

This table presents the marginal effects from a probit regression for a spouse's decision to hold term life insurance for the whole sample (Table III). Baseline is the spouse being male, 65 years old, and 3 years older than his partner (sample mean is 2.55 for husbands), having three children (sample mean is 3.2), and the same pension income as the other spouse and the household having sample mean log financial wealth (10.91) and log household income (10.23) without participating in the stock market. The corresponding probability for holding term life insurance is 32.48%. Marginal probabilities represent the respective absolute changes in the probability from being female, increasing the number of children by one, the age by 10 years or the age difference by 1 year, being the only pension income receiver in the household, a one standard deviation increase in log financial wealth (2.16), or log household income (0.53), as well as from participating in the stock market. The \( p \)-values account for cluster-robust standard errors. *, **, and *** indicate significance levels 10%, 5%, and 1%, respectively. Source: authors' calculations.

<table>
<thead>
<tr>
<th></th>
<th>Marginal probability (%)</th>
<th>( p )-value (significance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>-2.63</td>
<td>0.059*</td>
</tr>
<tr>
<td>Number of children</td>
<td>0.43</td>
<td>0.124</td>
</tr>
<tr>
<td>Age / 10</td>
<td>-2.61</td>
<td>0.000***</td>
</tr>
<tr>
<td>Age difference</td>
<td>0.12</td>
<td>0.457</td>
</tr>
<tr>
<td>Pension income share</td>
<td>4.20</td>
<td>0.000***</td>
</tr>
<tr>
<td>Log financial wealth</td>
<td>-2.69</td>
<td>0.000***</td>
</tr>
<tr>
<td>Log household income</td>
<td>3.52</td>
<td>0.000***</td>
</tr>
<tr>
<td>Stock participation</td>
<td>-3.10</td>
<td>0.004***</td>
</tr>
</tbody>
</table>
(10n − 5)th and the (10n + 5)th percentile. For each bin, we derive as input parameters for our model the average wealth to income ratio as well as the couples’ average income share from Social Security, husband’s company pensions, and wife’s company pensions. As it turns out, the relative importance of the different income sources is rather stable across the wealth distribution. Hence, for every bin, we use the overall sample average income source distribution and assume that 67% of the couple’s income results from Social Security with a survivor benefit of two-thirds, and 26% (7%) stem from the husband’s (wife’s) company pension without any survivor benefits.30 Based on these input parameters, as well as the calibration for our baseline couple from Section 3, we determine for each wealth bin the average share of liquid wealth that will be annuitized at the

Table V. OLS regression for (log) face values of term life insurance in the HRS

This table presents coefficients and marginal effect of an OLS regression on the (log) face values of term life insurance holdings for the subsample of term life insurance holders. The baseline case of a 65-year-old male spouse, being 3 years older than his partner (subsample mean is 2.60 for husbands), having three children (subsample mean is 3.2), and the same pension income as the other spouse and the household having subsample mean log financial wealth (10.74) and log household income (10.27) without participating in the stock market, has a face value of $33,300. The marginal effects are the relative changes in face values from being female, increasing the number of children by one, the age by 10 years or the age difference by 1 year, being the only pension income receiver in the household, a one standard deviation increase in log financial wealth (2.15), or log household income (0.53), as well as from participating in the stock market. The p-values account for cluster robust standard errors. *, **, and *** indicate significance levels 10%, 5%, and 1%, respectively. Source: authors’ calculations.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Marginal effects (%)</th>
<th>p-value (significance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>−0.632</td>
<td>−46.85</td>
</tr>
<tr>
<td>Number of children</td>
<td>−0.007</td>
<td>−0.72</td>
</tr>
<tr>
<td>Age / 10</td>
<td>−0.661</td>
<td>−48.37</td>
</tr>
<tr>
<td>Age difference</td>
<td>0.018</td>
<td>1.81</td>
</tr>
<tr>
<td>Pension income share</td>
<td>0.172</td>
<td>9.00</td>
</tr>
<tr>
<td>Log financial wealth</td>
<td>0.066</td>
<td>15.23</td>
</tr>
<tr>
<td>Log household income</td>
<td>0.177</td>
<td>9.76</td>
</tr>
<tr>
<td>Stock participation</td>
<td>−0.009</td>
<td>−0.89</td>
</tr>
<tr>
<td>Constant</td>
<td>12.070</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Abstracting from survivor benefits in company pensions is a conservative approach, as lower survivor benefits will tend to increase annuity demand. Hence, with company pensions providing survivor benefits by default, private annuity demand should be even lower than reported here.

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Table VI. Utility gains from access to the private annuity market

This table shows the annuity demand predicted by our model and the corresponding relative increase in the certainty equivalent of utility from having access to the private annuity market for various empirical wealth quantiles at the age of 65 years. The second column shows the corresponding financial wealth. The third column shows the mean wealth to income ratio for the quantiles (mean of subsample between +5% and −5% around the quantile). The fourth column reports the model-based optimal expenditures on annuities as a fraction of financial wealth in case the annuities are available only at the age of 65 years. The fifth column shows the corresponding increase in the certainty equivalent of utility at the age of 65 years gained by the access to the private annuity market. Source: authors’ calculations.

<table>
<thead>
<tr>
<th>Wealth decile (%)</th>
<th>Average financial wealth (in 2010 dollars)</th>
<th>Average wealth to income ratio</th>
<th>Expenditures on annuities (%)</th>
<th>Utility gained by annuity access (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1,658</td>
<td>0.10</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>20</td>
<td>13,455</td>
<td>0.59</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>30</td>
<td>37,361</td>
<td>1.65</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>40</td>
<td>68,725</td>
<td>2.70</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>50</td>
<td>114,604</td>
<td>3.78</td>
<td>2.3</td>
<td>0.00</td>
</tr>
<tr>
<td>60</td>
<td>191,302</td>
<td>6.62</td>
<td>25.1</td>
<td>0.35</td>
</tr>
<tr>
<td>70</td>
<td>322,093</td>
<td>10.99</td>
<td>40.5</td>
<td>1.32</td>
</tr>
<tr>
<td>80</td>
<td>498,142</td>
<td>19.11</td>
<td>50.8</td>
<td>3.00</td>
</tr>
<tr>
<td>90</td>
<td>792,221</td>
<td>26.34</td>
<td>54.6</td>
<td>4.15</td>
</tr>
</tbody>
</table>

beginning of retirement. As in our sensitivity analysis in Section 3.2, we also derive the couples’ utility increases from gaining access to the private annuity market.

Our findings are summarized in Table VI. Columns 1 and 2 present average financial wealth as well as the average wealth to income ratio within each bin. Financial wealth averages from $1,658 in the first decile group to $792,221 in the 9th decile bin. Average wealth to income ratios monotonically increase with financial wealth and range from 0.1 (first decile) to 26.34 (9th decile). Column 3 shows the average fraction of financial wealth optimally annuitized immediately upon retirement, whereas Column 4 presents the utility increases from having access to the private annuity market. For bins one through five, i.e., for more than half the sample population, demand for private annuities is virtually nonexistent or negligible. Those in or below the 4th wealth decile do not purchase any private annuities, whereas those in the 5th decile annuitize a mere 2.3% of the little initial wealth available. With income from Social Security and company pensions, these couples are already sufficiently annuitized. Consequently, for more than half of these older households having access
to private annuities does not result in measurable utility gains. With increasing financial means, however, the fraction of wealth immediately annuitized rises sharply. Those in the two highest deciles should optimally invest more than half of their liquid wealth in private annuities 50.8% and 54.6%, respectively. Yet, despite the apparently strong annuity demand within these groups, utility increases from having access to the private annuity market that amount to 4.15%, at best. This may explain why even wealthier households refrain from purchasing private annuities and, hence, why empirically observable annuity demands are negligible throughout the wealth distribution.

6. Conclusion

We present a lifecycle consumption and portfolio choice model for a retired couple with an intentional bequest motive and uncertainty about their joint lifetimes. In most lifecycle models, this uncertainty has two aspects. On the one hand, the household faces the risk of over consuming and hence outliving its assets. On the other hand, the household might under consume and leave an unintentionally high bequest. Explicitly modeling a couple, our model adds a 3rd aspect, which is that the early death of one spouse might result in an annuity income drop for the surviving partner that substantially exceeds the corresponding reduction in consumption needs. Against this background, we study how a couple can optimally hedge this risk by dynamically investing in payout annuities, term life insurance, stocks, and bonds.

Our results show that couples strive to achieve a symmetrical distribution of annuitized income between both spouses. Optimal annuity portfolios have a large share of joint annuities or, equivalently, high survivor benefits. If preexisting retirement income is asymmetrically distributed, then demand for term life insurance is high. This is due to the provision motive, i.e., the couple’s need to insure against the loss of the major earner’s survival contingent annuity income in case an early death. Consequently, having access to life insurance markets considerably enhances older couples’ welfare. To support a pure bequest motive, however, liquid wealth is generally preferred over term life insurance. Only in cases with very low liquid wealth when compared to annuitized retirement income, life insurance will be purchased to support intergenerational wealth transfer.

The model predictions accord well with empirical evidence. Econometric analyses based on HRS data for elderly US households indicate that the degree of pension income asymmetry has a significantly positive effect on the
probability of holding life insurance and on face values. Thus, empirical evidence support the predictions that term life insurance demand by retired couples is mostly driven by the desire to provide for a surviving spouse and not by their wish to bequeath wealth to their children or others.

Calibrating our model to the empirical distribution of financial wealth and pension income for retired US couples, we find that the utility costs of not integrating additional private life annuities into the retirement portfolio are low. This is due to the fact that many retired couples in the HRS data set have relatively low liquid wealth, while they are endowed with quite generous joint life annuities from Social Security. This may explain why so few households today purchase additional annuities in the private market and, hence, it may offer another explanation of the annuity participation puzzle. An interesting question, however, will be whether the anticipated reductions in Social Security benefits and the ongoing transition from traditional-defined benefit pension schemes to defined contribution plans will increase the appeal of private annuities for future retirees.

References


## Appendix

### Table A1. Detailed descriptive statistics for retired couples in the HRS

Sample statistics of key household quantities for different age groups (age of husband) of the HRS waves 5–10. Dollar values are inflated to 2010 dollars. The sample consists of married or partnered households in which the husband is at least 65 years old, at least one spouse receives social security retirement, and none receive any labor income. To avoid outliers, households with negative financial wealth as well as the top 5% both in financial wealth and household income distribution are excluded. Financial wealth-related stock fractions include stocks, mutual funds, and IRAs, assuming IRA stock exposures of 0%/16%/31%/52% for the lowest/second/third/highest wealth quartile (see Waggle and Englis, 2000; Table II). Term life insurance participation equals one, if life insurance holdings but no endowment plans are reported, zero otherwise. The average term life insurance and endowment plans face values are conditioned on the respective holdings. Pension participation equals one, if positive pension income is reported, zero otherwise. “Total income” is the sum of social security income and pension income. Income shares of husband is the husband’s respective income divided by the sum of both husband’s and wife’s respective income.

<table>
<thead>
<tr>
<th>Age group (age of husband)</th>
<th>Mean (SD)</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>65–69</td>
<td>70–74</td>
</tr>
<tr>
<td>Financial wealth</td>
<td>219,310 (270,199)</td>
<td>228,794 (295,984)</td>
</tr>
<tr>
<td>Total household income</td>
<td>31,885 (17,087)</td>
<td>33,380 (15,129)</td>
</tr>
<tr>
<td>Household income from social security</td>
<td>18,577 (7,475)</td>
<td>21,129 (7,032)</td>
</tr>
<tr>
<td>Household income from pensions</td>
<td>13,308 (15,506)</td>
<td>12,251 (14,204)</td>
</tr>
<tr>
<td>Wealth/Income ratio</td>
<td>8.15 (13.83)</td>
<td>7.12 (10.23)</td>
</tr>
<tr>
<td>Stock participation</td>
<td>0.46 (0.50)</td>
<td>0.44 (0.50)</td>
</tr>
<tr>
<td>Stock fraction (conditional on participation)</td>
<td>0.48 (0.25)</td>
<td>0.49 (0.25)</td>
</tr>
<tr>
<td>Term life participation—husband</td>
<td>0.36 (0.48)</td>
<td>0.35 (0.48)</td>
</tr>
<tr>
<td>Term life participation—wife</td>
<td>0.34 (0.47)</td>
<td>0.29 (0.45)</td>
</tr>
<tr>
<td>Term life face value—husband</td>
<td>65,323 (120,480)</td>
<td>46,475 (93,493)</td>
</tr>
<tr>
<td>Term life face value—wife</td>
<td>37,828 (68,121)</td>
<td>29,297 (83,195)</td>
</tr>
<tr>
<td>Endowment plan participation—husband</td>
<td>0.41 (0.49)</td>
<td>0.39 (0.49)</td>
</tr>
<tr>
<td>Endowment plan participation—wife</td>
<td>0.32 (0.47)</td>
<td>0.3 (0.45)</td>
</tr>
<tr>
<td>Endowment plan face value—husband</td>
<td>50,166 (72,813)</td>
<td>32,418 (50,005)</td>
</tr>
<tr>
<td>Endowment plan face value—wife</td>
<td>50,166 (72,813)</td>
<td>32,418 (50,005)</td>
</tr>
<tr>
<td>Private pension participation of husband</td>
<td>0.32 (0.47)</td>
<td>0.3 (0.45)</td>
</tr>
<tr>
<td>Private pension participation of wife</td>
<td>0.28 (0.45)</td>
<td>0.32 (0.47)</td>
</tr>
<tr>
<td>Total income share of husband</td>
<td>0.72 (0.22)</td>
<td>0.69 (0.16)</td>
</tr>
<tr>
<td>Social security income share of husband</td>
<td>0.69 (0.23)</td>
<td>0.64 (0.16)</td>
</tr>
<tr>
<td>Pension income share of husband</td>
<td>0.76 (0.36)</td>
<td>0.77 (0.35)</td>
</tr>
</tbody>
</table>