Risk Management and Saving:
On Income Effects and Background Risk

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Abstract

We study the interplay of intertemporal risk management and saving decisions. We define risk management broadly by allowing the activity to influence the severity of loss, the probability of loss, or both simultaneously. Due to the similar cost-benefit structure of risk management and saving decisions a substitution effect arises whose implications are analyzed for changes in income and background risk. Typically, the direct effects of exogenous variations on risk management and saving decisions go in the same direction but because of substitution net effects become a priori ambiguous. We resolve this ambiguity by deriving necessary and sufficient conditions to sign the comparative statics. Our paper highlights fundamental differences between single- and multi-instrument models which have broad implications for empirical and theoretical work.

Keywords: risk management · saving · income effects · background risk · substitution.

JEL-Classification: D81 · D90 · D91 · E21

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1 Introduction

In the face of risk individuals will invest in risk management to mitigate the occurrence and severity of untoward events. A classical distinction has been made between self-insurance or loss reduction, which reduces the severity of loss, and self-protection or loss prevention, which reduces the probability of loss (Ehrlich and Becker, 1972). Whereas the first activity is a substitute for insurance, the latter can substitute or complement insurance coverage. Furthermore, more risk-averse agents in the sense of Pratt (1964) invest more in self-insurance but not necessarily in self-protection (Dionne and Eeckhoudt, 1985; Briys and Schlesinger, 1990; Jullien et al., 1999). Also wealth or income effects are different for the two forms of risk mitigation (Sweeney and Beard, 1992; Lee, 2005). As a result, self-insurance and self-protection decisions have often been analyzed separately.

As pointed out by Lee (1998), risk management may affect both the probability and severity of loss simultaneously. We use this more general notion and put it into an intertemporal context. To mitigate (anticipated) future consumption risks, agents invest today in a costly activity that increases expected future consumption utility. This intertemporal view on risk management decisions has gained momentum in recent years. However, as soon as we apply it, alternative means of responding to future consumption risk are at hand.

Saving shifts wealth from today into the future and makes a given consumption risk easier to bear for a risk-averse individual. As such it appears as a natural substitute for any risk management activity. Indeed, both instruments trade off current consumption against an increase in expected future consumption. We show that this substitution effect holds not only for self-protection (Menegatti and Rebessi, 2011) but also for the generic class of risk management activities considered in this paper. It is a universal phenomenon whose implications for

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1 This also holds outside the realm of expected utility as established by Courbage (2001) within Yaari’s (1987) dual theory of choice.

2 Engaging a good lawyer reduces the probability of conviction and the punishment for crime. Proper vehicle maintenance and cautious driving reduce the likelihood of an accident and the magnitude of loss in case an accident occurs. Regular medical checkups decrease the probability and severity of future illnesses.


4 Drèze and Modigliani (1972) study consumption decisions when agents are exposed to risk and the comparative effects of intertemporal preferences. Kimball (1990) points out that prudence, i.e., a positive third derivative of utility, is the necessary and sufficient condition for future income uncertainty to induce precautionary wealth accumulation. Bommier et al. (2012) derive general results on the impact of risk aversion on saving motives thereby clarifying the link between prudence and risk aversion.
income effects and background risk are documented in this article. As a consequence, a sharp line needs to be drawn between isolated risk management and saving decisions and combined ones. Without this distinction, spurious predictions will be made which can lead to difficulties in reconciling theory with empirical results.

To give an example, many countries have or had tax subsidized saving programs in place (Engelhardt, 1996; Bovenberg, 1989). An increase in the tax subsidy makes saving more attractive. Still, empirical results on the behavioral effects of Individual Retirement Accounts were mixed (Venti and Wise, 1990; Gale and Scholz, 1994). Our results suggest that a negative substitution effect arising from risk management activities may outweigh positive effects on saving so that overall savings do not increase or might even decrease upon an exogenous change. Furthermore, the (unobservable) use of risk management activities and heterogeneity therein represent a potential channel for explaining heterogeneity in saving responses. As a result, even if no significant increase in savings can be observed, this does not necessarily imply that the program under consideration is to be judged ineffective. Furthermore, a null response does not necessarily constitute a contradiction between theory and empirical evidence.

The same applies to the empirical study of risk management decisions. Incentivizing safety investments can lead to less risk management rather than more if the substitution effect from saving is dominant. In short, as soon as risk management and saving are considered simultaneously to optimize expected intertemporal consumption utility, the comparative statics are governed by the substitution effect which implies the ambiguity of the majority of exogenous changes. Resolving this ambiguity requires knowledge about the structure of the underlying decision problem implying that there are no generic conditions that can be applied universally. This represents a major challenge for empirical work that aims at measuring behavioral responses in saving or risk management following exogenous variations.

In the next section we develop a simple model of risk management and saving. We discuss comparative statics which are straightforward and unambiguous for risk management but become ambiguous in the presence of saving. Section 3 introduces background risk and shows how the optimal risk management and saving portfolio reacts to changes in the size of a background risk. In section 4, we extend the result that risk management and saving are substitutes to various contexts. We show that it is a universal phenomenon which is present in a huge variety of potential model extensions. The last section concludes.
2 A Simple Model

2.1 Notation

We consider a decision-maker (DM) who lives for two periods, \( t_1 \) and \( t_2 \), now and then. She receives income \( w_i \) in \( t_i \) where we assume at the moment that first-period income is risk-free. However, second-period income is subject to a random loss of \( l \) that occurs with probability \( p \).

Risk and time preferences are characterized by the first-period utility function \( U \), the second-period utility function \( V \), and a utility discount factor of \( \beta \). \( U \) and \( V \) are assumed to be increasing and concave which reflects the DM’s non-satiation and risk aversion. Consequently, expected utility of intertemporal consumption is given by

\[
EU = U(w_1) + \beta [pV(w_2 - l) + (1 - p)V(w_2)].
\]

(1)

2.2 Risk Management

To reduce the second-period consumption risk individuals can pursue a costly risk management activity. We denote by \( e \) the intensity of this activity (“effort”) and by \( c(e) \) the cost associated with it. Naturally, \( c(0) = 0 \) and \( c' > 0 \), and we also assume that \( c'' \geq 0 \). Furthermore, we make the assumption that effort may affect the probability of loss \( (p' \leq 0) \) and/or the severity of loss \( (l' \leq 0) \) with at least one inequality strict. This modeling is very flexible as it comprises self-protection and self-insurance decisions (Ehrlich and Becker, 1972) as well as combinations of the two, so called self-insurance-cum-protection decisions (Lee, 1998).

Hence, the individual’s objective function is given by

\[
\max_e EU(e) = U(w_1 - c(e)) + \beta [p(e)V(w_2 - l(e)) + (1 - p(e))V(w_2)].
\]

(2)

Throughout the entire paper we consider interior solutions that can be characterized via the first-order conditions. For (2) we obtain

\[
EU_e = -c'U_1' + \beta p'[V_2L - V_2N] - \beta pl'V_2' = 0,
\]

(3)

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5 As a special case of self-insurance also market insurance decisions are covered in our model, i.e. a coinsurance contract for which the premium and the retained loss are linear functions of coverage.

6 Parameter subscripts denote partial derivatives, e.g., \( EU_e = \partial EU/\partial e \).
where we denote \( U_1 := U(w_1 - c(e)) \), \( V_{2L} := V(w_2 - l(e)) \), and \( V_{2N} := V(w_2) \) to compress notation.\(^7\) Note that the marginal cost from investing in risk management is given by the first term as measured in utility units, i.e., marginal expenditures on risk management times marginal utility of first-period consumption. The marginal benefit consists of the second term, which captures the effect of a reduction of the probability of loss on expected utility, and the third term, which measures the impact of a less severe loss on expected utility.

Comparative statics are straightforward and are summarized in the following proposition.\(^8\)

**Proposition 1.** The optimal level of risk management is

a) increasing in first-period income,

b) decreasing in second-period income,

c) and increasing in the utility discount factor.

**Proof.** All proofs can be found in the appendix.

The intuition is the following. If there is more income at the disposal of the DM in the first period, expenditures on risk management will hurt less in terms of forgone consumption, i.e., the marginal cost is lower. The marginal benefit, however, remains unchanged implying a positive net effect. An increase in second-period income lowers the marginal benefit from risk management. Diminishing marginal utility leads the utility difference between no-loss state and loss state to shrink as income increases, therefore the first component of the marginal benefit is decreased. The same is true for the second component of marginal benefit, i.e., more income makes a loss of the same size easier to bear. Both effects combined induce the intensity of risk management to decrease, as the marginal cost are unaffected by a change in second-period income. When the utility discount factor increases, this means that the DM becomes more patient. As a result the marginal benefit of both the reduced probability and severity of loss is

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\(^7\) Under standard assumptions on the cost \((c' > 0, c'' \geq 0)\) and the risk management technology \((p' \leq 0, p'' \geq 0, l' \leq 0, l'' \geq 0)\), the objective function is concave in the intensity of risk management,

\[-c''U'_1 + (c')^2U''_1 + \beta p''(V_{2L} - V_{2N}) - 2\beta p' l' V_{2L} - \beta p'' l'' V_{2L} + \beta p(l')^2 V_{2L} < 0.\]

Whereas in single-period models the dependence of \( p \) on \( e \) creates non-concavities in expected utility (Shavell, 1979), this does not happen in our case.

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\(^8\) Besides income or wealth effects researchers have investigated the effect of risk preferences on risk management. In the last 10 years, the link between prudence or downside risk aversion and self-protection has received considerable attention (Eeckhoudt and Gollier, 2005; Chiu, 2005; Menegatti, 2009; Dionne and Li, 2011; Peter, 2013). The reason is that risk aversion alone does not explain self-protection well (Dionne and Eeckhoudt, 1985).
larger whereas the marginal cost is unaffected. This is because expenditures on risk management precede its effect. Consequently, the net effect on risk management is positive.

2.3 Risk Management and Saving

As outlined in the introduction, another instrument that impacts intertemporal consumption is the accumulation of savings. Let \( s \) be the DM’s savings in the first period\(^9\) with \( r \geq 0 \) being the interest rate which we assume to be deterministic for simplicity. The individual’s objective function changes to

\[
\max_{e, s} EU = U(w_1 - c(e) - s) + \beta [p(e)V(w_2 - l(e) + (1 + r)s) + (1 - p(e))V(w_2 + (1 + r)s)]. \tag{4}
\]

The first-order conditions read as\(^\text{10}\)

\[

EU_e = -c'u_1 + \beta p'[V_{2L} - V_{2N}] - \beta pl'V_{2L} = 0, \tag{5}
\]

\[
EU_s = -u_1 + \beta (1 + r) [pV_{2L} + (1 - p)V_{2N}] = 0. \tag{6}
\]

The first condition solves the trade-off between marginal cost of investing in risk management and marginal benefit from such investment. The second condition is the usual consumption smoothing condition over the life-cycle, i.e., expected marginal utility of consumption must be equal over both periods. By equating marginal cost we see that at the optimum a dollar has the same marginal expected utility benefit whether it is spend on risk management or saving. If both instruments are considered simultaneously, their interaction will be non-trivial as indicated in the following remark.

Remark 1. There is a substitution effect between risk management and saving.

This remark shows that the substitution effect between self-protection and saving (Menegatti and Rebessi, 2011) and self-insurance and saving (Hofmann and Peter, 2014) carries over to our general notion of risk management decisions.\(^\text{11}\) The reason is that with higher savings there is less first-period income which increases the marginal cost of the risk management activity

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\(^9\) If \( s \) is negative, this indicates borrowing against future consumption to finance current consumption.

\(^\text{10}\) We assume that the second-order conditions are satisfied. Note that \( EU_{ee} < 0 \) as in Footnote 7 and that \( EU_{ss} = U''_1 + \beta(1 + r)^2 [pV_{2L} + (1 - p)V_{2N}] < 0 \) due to risk aversion. Hence, we assume for a maximum that the determinant of the Hessian of \( EU \) be positive, \( D = EU_{ee}EU_{ss} - EU_{es}^2 > 0 \).

\(^\text{11}\) Furthermore, Dionne and Eeckhoudt (1984) find that insurance and saving are substitutes in the Hicksian sense; we explicitly allow for income effects, i.e., we study Walrasian demand for risk management and saving.
(see Proposition 1a). Also, with higher savings there is more second-period income which decreases the marginal benefit of risk management (see Proposition 1b). It pays less to reduce the probability or the severity of loss when the DM is richer due to diminishing marginal utility. Consequently, the net effect is negative so that an exogenous increase in saving reduces the level of risk management and vice versa.

This substitution effect renders the comparative statics ambiguous. Consider an increase in first-period income; first, the marginal cost of the risk management activity is lower due to the fact that marginal utility is diminishing. The marginal benefit of risk management is unaffected when current income is increased. Therefore, we would expect an increase in the intensity of risk management (see Proposition 1a). Due to the substitution effect this would induce less savings. However, with more first-period income also saving is more attractive, as the marginal cost are lower, whereas the marginal benefit is unaffected. This would lead to an increase in saving. Hence the direct effect and the indirect or substitution effect on saving point in opposite directions so that the net effect is ambiguous. The same reasoning holds for the level of risk management and the other exogenous parameters.

Consequently, it depends on the comparative efficiency of risk management and saving whether the direct or indirect effects prevail as a reaction to changes in exogenous parameters. We denote by $MB^i$ ($MC^i$) the marginal benefit (marginal cost) of the risk management activity ($i = e$) and of saving ($i = s$), respectively. With this notation, we obtain the following conditions

$$MB^e_s > cMB^s_s.$$  
(7)

$$c^i (MB^s_e - MC^e_s) > MB^e_c - MC^e_c.$$  
(8)

$$\frac{1}{MB^s_e} (MB^s_e - MC^e_s) > \frac{1}{MB^s_c} (MB^e_c - MC^e_c).$$  
(9)

With higher second-period income it pays less to invest in risk management and to accumulate savings due to diminishing marginal utility. The intuition behind condition (7) is that this effect is weaker for risk management than for saving so that risk management is more efficient from this perspective. Condition (8) makes a different comparison. The marginal benefit of saving decreases with more risk management whereas its marginal cost increase. This is due to the

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12 In detail, we have that $MB^e = \beta p' [V_{2L} - V_{2N}] - \beta p V'_{2L}$, $MB^s = \beta(1 + r) [pV'_{2L} + (1 - p)V'_{2N}]$, $MC^e = c'U'_1$, $MC^s = U'_1$, which is obtained from the first-order conditions (5) and (6).
substitution effect (see Remark 1). Furthermore, also the marginal benefit of risk management decreases with more risk management whereas its marginal cost increase. This is due to the second-order condition for risk management (see Footnote 7). Condition (8) states that the net effect of an increase in risk management on saving is smaller than that on risk management itself so that saving is comparatively more efficient. The last condition is the most complex one. Again, it compares how the marginal benefit and the marginal cost of risk management and saving are affected by marginal variations in the risk management decision, additionally controlling for the impact of saving on the marginal benefit of each action. The net effect on saving of the changes under consideration is smaller than the net effect on risk management so that saving is comparatively more efficient in this sense.

It is noteworthy that these conditions are neither necessary nor sufficient for one another. Said differently, knowing that one of the three holds it is not possible to conclude that one of the other two holds. Also, additional simplifications can be obtained by making the assumption that the cost of risk management are measured in currency, i.e. that $c(e) = e$. Then, obviously $c'(e) = 1$ and $MC^s = MC^e = U_1'$. Conditions (7) to (9) are useful to sign the comparative statics results for optimal risk management and saving decisions. We summarize the results in the following proposition.

**Proposition 2.**

a) The optimal level of risk management is increasing in first-period income, increasing in second-period income, and increasing in the utility discount factor if and only if condition (7) is satisfied.

b) The optimal level of saving is increasing in first-period income and increasing in the utility discount factor if and only if condition (8) is satisfied; it is increasing in second-period income if and only if condition (9) is satisfied.

As outlined above, in the model with two decisions there is a direct effect (see Proposition 1) and an indirect or substitution effect (see Remark 1), which point into opposite directions. For instance, the direct effect of more first-period income on risk management is positive but the substitution effect is negative due to the fact that also the direct effect on saving is positive. If risk management is the more efficient way to react to exogenous changes in the environment,

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13 One example would be the case where $c(e)$ denotes the premium for insurance coverage.
there will be an increase. A priori it is not clear how to express comparative efficiency in our setting but the answers are given by conditions (7) - (9).¹⁴

Let us elaborate on the comparative efficiency conditions by looking at a variation of first-period income. First, note that an increase in first-period income only affects the marginal cost of risk management and saving, respectively. Increasing income in the first period lowers the marginal cost of risk management and saving inducing a positive direct effect on both instruments. In case of risk management, this positive effect is measured as the marginal impact of first-period income on the first-order condition for optimal risk management times the negative of the marginal impact of saving on the first-order condition for optimal saving, as obtained from the implicit function rule. The negative substitution effect from saving on risk management is obtained as the marginal impact of first-period income on the first-order condition for optimal saving times the cross-derivative of expected intertemporal consumption utility. By drawing on the fact that a marginal increase of first-period income has the same effect on the marginal cost of the two instruments as a marginal decrease of saving, we can focus on the marginal benefits of the two instruments. Furthermore, we can utilize the fact that $c'$ can be interpreted as a “conversion rate” between the marginal cost of saving and the marginal cost of risk management. This is how condition (7) is obtained.

The remaining cases can be treated similarly. Let us highlight the fact that conditions (7) - (9) involve individual preference traits (as captured by $U, V,$ and $\beta$), properties of the instruments available (as measured by $r, c, p,$ and $l$), and the two exogenous income levels (as measured by $w_1$ and $w_2$). As such they are specific to the DM under scrutiny and the instruments that are used to determine the intertemporal consumption pattern. We cannot expect universal conditions that would result in unambiguous predictions for the comparative statics in a large set of applications. As a consequence the empirical identification of changes in risk management and saving decisions is challenging.

¹⁴ Our results are consistent with earlier results (Menegatti and Rebessi, 2011) and show that the effects carry over to our more general specification of risk management decisions.
3 The Effect of Background Risk

3.1 Preliminaries

When mitigating risks DMs are often confronted with other sources of risk they cannot control. These other risks have been labelled “background risk” in the literature. One example is income risk which is assumed to be independent of a property risk. The existence of background risks has important consequences for optimal decision making.\textsuperscript{15}

In this section, we will analyze the impact of background risk on optimal risk management decisions. The literature that explicitly models the intertemporal character of risk mitigation is still nascent so that existing results which mainly focus on single-period models cannot be applied. The results on isolated risk management decisions are consistent with the more specific set-up of Courbage and Rey (2012), who focus on self-protection. We also combine risk management and saving decisions and show how generalized versions of the conditions developed in subsection 2.3 can be utilized to sign the various effects. Also here the substitution effect is decisive throughout the analysis.

3.2 Risk Management and Background Risk

Background risk may be present at different points in time. Returning to the individual’s objective (2) for optimal risk management, a general way to introduce background risk would be

$$\max_{e} \mathbb{E}U (w_1 - c(e) + \tilde{\varepsilon}_1) + \beta [p(e)\mathbb{E}V (w_2 - l(e) + \tilde{\varepsilon}_2) + (1 - p(e))\mathbb{E}V (w_2 + \tilde{\varepsilon}_2)].$$

(10)

For simplicity we assume the risks $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ to be independent of the risk of loss and to have zero mean. In the following analysis we will distinguish between different cases.

- First-period background risk (FPBR) implies that $\tilde{\varepsilon}_2 \equiv 0$.

- Second-period background risk (SPBR) implies that $\tilde{\varepsilon}_1 \equiv 0$.

\textsuperscript{15} For instance, health risks (Rey, 2003) or the risk of contract non-performance have been interpreted as types of background risk (Doherty and Schlesinger, 1990; Hau, 1999). To answer questions of how the introduction of background risk (Gollier and Pratt, 1996) and changes in background risk (Eeckhoudt et al., 1996) affect risk-taking behavior towards endogenous risks typically higher-order risk preferences need to be considered.
Note that SPBR is not conditional on the state of nature that obtains.\footnote{Fei and Schlesinger (2008) study so-called state-dependent background risk. The motivation is that income uncertainty might take different forms conditional on whether a loss obtains or not. Our results can easily be extended to a model of state-dependent background risk.} We briefly mention the effect of introducing a background risk in the following remark. We draw on the notion of prudence (Kimball, 1990) which is equivalent to downside risk aversion (Menezes et al., 1980).\footnote{Eeckhoudt and Schlesinger (2006) characterize prudence as a location preference for adding a zero-mean risk to the better of two equiprobable states rather than to the worse. Descriptively, this preference trait appears to be quite prevalent (e.g., Ebert and Wiesen, 2011).}

**Remark 2.**

a) If first-period utility is prudent ($U''' > 0$), the introduction of FPBR decreases the optimal level of risk management.

b) If second-period utility is prudent ($V''' > 0$), the introduction of SPBR increases the optimal level of risk management.

The intuition behind our findings is straightforward. For a prudent DM, FPBR increases the marginal cost of risk management whereas the marginal benefit is unaffected. As a result, the intensity of risk management decreases so that current consumption increases whereas expected future consumption decreases. This can be interpreted as a precautionary effect because the individual shifts wealth to the period where it bears the background risk by reducing expenditures on risk management. For SPBR two effects can be observed. First, expected marginal utility in the loss-state increases and as a consequence the marginal benefit of reducing the severity of loss increases. Second, when future income becomes risky the utility difference between high- and low-wealth states increases for prudent DMs. Hence, also the marginal benefit of reducing the probability of loss increases. The net effect is, of course, positive so that a precautionary increase in the intensity of risk management takes place.

Note the similarity to isolated saving decisions again. Also in the case of saving the introduction of FPBR implies a precautionary accumulation of wealth in the period where the background risk appears. Consequently, savings are reduced to increase current consumption. Similarly, as pointed out by Kimball (1990), the introduction of SPBR induces precautionary wealth accumulation in the second period; as a consequence, savings increase. In Proposition 1 we studied changes in income which is a first-order effect. To elaborate on the second-order effects of background risk we assume that $\tilde{\epsilon}_1 = k \cdot \tilde{\epsilon}$ for the first period and $\tilde{\epsilon}_2 = k \cdot \tilde{\epsilon}$ for the second period, with scale parameter $k > 0$ and $\tilde{\epsilon}$ being a zero-mean risk. A marginal increase in
Proposition 3.

(a) If first-period utility is prudent ($U'''' > 0$), an increase in FPBR decreases the optimal level of risk management.

(b) If second-period utility is prudent ($V'''' > 0$), an increase in SPBR increases the optimal level of risk management.

The intuition is very similar to before. Prudence implies that background risk raises the marginal value of income. As a result the individual shifts wealth to the period in which the background risk has become riskier. If this is the first period, risk management needs to be reduced to save on risk management costs. If it is the second period, risk management needs to be increased to have more wealth in the bad state and to face the bad state with a lower probability. Similar results with respect to saving were demonstrated by Eeckhoudt and Schlesinger (2008).

3.3 Risk Management, Saving, and Background Risk

As a next step we include saving as a means of optimizing expected intertemporal consumption utility. As mentioned before, Kimball (1990) was the first to show that the introduction of future income risk leads to the accumulation of precautionary savings if and only if the DM is prudent. As shown in Remark 1 besides the direct effects of background risk on risk management and saving decisions a substitution effect arises according to the similar cost-benefit structure of the two instruments.

In the face of background risk, the individual’s objective function when both risk management and saving are available reads as

$$\max_{e, s} \left\{ \mathbb{E}U \left( w_1 - c(e) - s + \tilde{\varepsilon}_1 \right) + \beta \left[ p(e) \mathbb{E}V \left( w_2 - l(e) + (1 + r)s + \tilde{\varepsilon}_2 \right) + (1 - p(e)) \mathbb{E}V \left( w_2 + (1 + r)s + \tilde{\varepsilon}_2 \right) \right] \right\}. \quad (11)$$

Paralleling the previous subsection we investigate background risks that might occur at different points in time. We start with FPBR. To use standard comparative statics techniques, we use

18 Furthermore, the intensity of prudence as measured by the coefficient of absolute prudence $-V''''/V''$ is monotonically related to the strength of the precautionary saving motive.
the previous decomposition of $\tilde{\epsilon}_1 = k \cdot \tilde{\epsilon}$ and vary the size of the background risk by considering changes in the scale parameter $k$. This sheds light on how the intensity of risk management and saving react to increases in the riskiness of background risk.

As in subsection 2.3 the comparative efficiency of risk management and saving has to be considered to determine the direction of the resulting effects when the size of a background risk changes. In the case of FPBR, we can “recycle” condition (7) from above and need the following extension of condition (8):

$$c' \left( MB_e^b - \mathbb{E} \tilde{MC}_e^b \right) > MB_e^e - \mathbb{E} \tilde{MC}_e^e. \tag{12}$$

Here $\tilde{MC}$ denotes marginal cost in the presence of background risk. Note that if condition (8) holds for all income levels $w_1$ it follows that (12) is satisfied because taking expectations preserves the inequality. We obtain the following proposition.

**Proposition 4.a.** If first-period utility is prudent ($U''' > 0$), an increase in FPBR decreases the optimal level of risk management if and only if (7) is satisfied, and decreases the optimal level of saving if and only if (12) is satisfied.

This reveals that background risk in the first period has an effect on the DM’s optimal decisions that is closely related to the effect of decreasing first-period income (see Proposition 2 a) and b)). The intuition for this result can be obtained by using the notion of the precautionary premium (Kimball, 1990). It is defined as the sure wealth reduction implying the same marginal utility as a zero-mean risk and is positive for prudent agents. Consequently, the effect of increasing the size of FPBR on marginal utility is identical to a sure reduction of first-period income when the DM is prudent. As a result the marginal cost of both risk management and saving increase whereas marginal benefits remain unaffected. Hence, there is a negative direct effect on both instruments and – as a consequence – a positive substitution effect (Remark 1) from each instrument on the other. Condition (7) informs about when the direct effect dominates for risk management and condition (12) about when this is the case for saving.

The case of SPBR is more involved. Unlike in the previous case, here not only marginal utility but also utility is affected, or more precisely, the utility premium (see Eeckhoudt and Schlesinger, 2009) when it comes to evaluating the marginal benefit of a reduction in probability

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19 Alternatively, it can be interpreted as the risk premium in the sense of Pratt (1964) for the zero-mean risk and the utility function given by the negative of marginal utility.
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of loss. Due to the fact that the precautionary premium and the risk premium associated with a background risk need not be identical, changes in the riskiness of SPBR are not isomorphic to a decrease in second-period income.\(^{20}\) Rather we need to take into account this differential effect on the marginal benefits of the two instruments considered as part of the comparative efficiency conditions. Technically we derive the following inequalities:

\[
MC_s \cdot \text{Cov} \left( \tilde{\varepsilon}, \tilde{MB}_e - c'\tilde{MB}_s \right) > \text{Cov} \left( \tilde{MB}_e, \tilde{MB}_s \right) - \text{Cov} \left( \tilde{MB}_s, \tilde{MB}_e \right) \quad (13)
\]

\[
\frac{1}{\text{Cov} \left( \tilde{\varepsilon}, \tilde{MB}_s \right)} \left( \frac{\text{E} \tilde{MB}_e - MC_e}{\text{Cov} \left( \tilde{\varepsilon}, \tilde{MB}_s \right)} \right) > \frac{1}{\text{Cov} \left( \tilde{\varepsilon}, \tilde{MB}_s \right)} \left( \frac{\text{E} \tilde{MB}_e - MC_e}{\text{Cov} \left( \tilde{\varepsilon}, \tilde{MB}_s \right)} \right). \quad (14)
\]

As before the tilde indicates the presence of the background risk. The left-hand side of condition (13) is related to condition (7) for the comparative efficiency when second-period income changes. However, the right-hand side picks up the effect that background risk does not affect utility and marginal utility in the same way and measures the differential impact. (14) is a modification of (9) to account for background risk. This gives rise to the following proposition.

**Proposition 4.b.** If second-period utility is prudent \(V'''' > 0\), an increase in SPBR increases the optimal level of risk management if and only if (13) is satisfied, and increases the optimal level of saving if and only if (14) is satisfied.

Although conditions (12) - (14) are quite different from conditions (7) - (9), we can reiterate the observation that individual preference traits, properties of the available instruments, and properties of the exogenous income distributions determine the comparative attractiveness of risk management and saving when it comes to reacting to changes in background risk. A priori it is not clear whether direct or indirect effects prevail and the conditions to resolve this indeterminacy are highly context-specific.

### 4 Generalizations and Robustness

#### 4.1 Preliminary Remarks

The central finding of this paper is that risk management and saving are substitutes in a very general sense. As shown in the previous two sections, this substitution effect involves trade-offs for income effects and background risks, i.e., for first- and second-order changes in the

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\(^{20}\) An exception is CARA utility for which the risk premium and the precautionary premium coincide for a given independent risk exposure.
distribution of income. Still, our model uses some simplifying assumptions that we address in this section. More specifically, we use two states of the world to model the potential loss, we restrict risk management to reducing expected loss, we eliminate technological uncertainty for the instruments under scrutiny, and we assume separability of consumption utility across points in time. We show that our main finding can still be considerably generalized such that these simplifications were mainly made for ease of exposition.

4.2 Multiple States of the World

It is common in the prevention literature to simplify the risk of loss by assuming two states of the world, a loss state and a no-loss state. Lee (1998) provides an extension of so-called self-insurance-cum-protection decisions to multiple states of the world. We modify his approach and adopt it to our setup. Let losses be given by random variable \( \tilde{l} = \tilde{l}(e) \) with cumulative distribution function \( F(e) \). Risk management is assumed to be beneficial as it reduces the size of a loss conditional an a given state of nature; this is \( l_e(x) \leq 0 \) where \( x \) represents the state of the world. Furthermore, we include that \( F_e \geq 0 \) which means that an increase in risk management implies a first-order stochastic deterioration in the distribution of losses. Roughly speaking, probability mass is shifted to the left so that the probability that the realized loss be less than a threshold becomes larger whatever the threshold. As losses are subtracted from future income, this implies a first-order stochastic improvement of future income.

With these specifications, the DM’s objective function becomes

\[
\max_{e,s} EU = U(w_1 - c(e) - s) + \beta \int V(w_2 - l(e, x) + (1 + r)s) dF(e, x).
\] (15)

The relationship between the intensity of risk management and saving is governed by the cross derivative of expected utility. In the case considered here, it is given by

\[
EU_{es} = c'U'' + \beta(1 + r) \int V'\,dF_e(e, x) - \beta(1 + r) \int V''l_e\,dF(e, x).
\] (16)

The first summand is negative because an increase in saving increases the marginal cost of risk management. Also the second is negative because \( e \) orders the distributions according to first-order stochastic dominance. Due to risk aversion, marginal utility is diminishing so that first-order stochastic improvements are disliked. Intuitively, with higher income in the second period it pays less to reduce the likelihood of losses. Finally, also the last term is negative due
to risk aversion. Consequently, the substitution between risk management and saving carries over to the case of multiple state of the world. The next step is to investigate the higher order case.

### 4.3 Nth Order Stochastic Dominance

Rather than restricting our attention to first-order stochastic improvements in the distribution of losses, we can also investigate the general case of Nth order stochastic dominance (Ekern, 1980).\(^{21}\) To this end we assume that the cumulative distribution functions are ordered by the intensity of risk management variable \(e\),

\[
\forall e_1 > e_2 : F(e_1) \succ_N F(e_2),
\]

with \(\succ_N\) denoting Nth order stochastic dominance. According to Ekern (1980) we know that every DM with \((-1)^k V^{(k)}(w) < 0\) for \(k = 1, \ldots, N\) will unanimously prefer \(F(e_1)\) to \(F(e_2)\). Under the assumption that \((-1)^k V^{(k)}(w) < 0\) for \(k = 1, \ldots, N + 1\) we can recoup the finding that risk management and saving exhibit a substitution effect.\(^{22}\) Inspecting (16) reveals that the first and the third term are unaffected by moving to the higher-order case. For the middle term we can exploit the fact that the subsequent derivatives of function \(V'\) change sign up to the \(N\)th derivative but start with a minus so that \(N\)th order stochastic improvements are disliked. This indicates that the benefit from \(N\)th order stochastic improvements is smaller for higher income levels, i.e., when savings are larger.

### 4.4 Technological Uncertainty

Another assumption is that the productivity of risk management activities and the return on saving are known with certainty in the first period. Authors have studied the impact of non-reliability of risk management decisions (e.g., Hiebert, 1989; Briys et al., 1991) as well as uncertain interest rates (e.g., Chiu et al., 2012). Let us replace \(e\) by \(e\tilde{\xi}\) with \(\tilde{\xi}\) distributed in \([0, 1]\), and \(r\) by \(\tilde{r} \geq -1\) in the intertemporal utility objective. The idea is that risk management might not be as effective as envisioned and that at the time of investing the return on saving might not be known for sure.

\(^{21}\) Jindapon and Neilson (2007) propose a model where the effects of risk management are not restricted to the first order.

\(^{22}\) This assumption is consistent with so-called mixed risk aversion (Caballé and Pomansky, 1996).
Under these assumptions the cross-derivative of expected consumption utility with respect to risk management and saving is given by

\[ EU_{es} = c' U''_1 + \beta E \xi p' (1 + \tau) \left( \tilde{V}'_{2L} - \tilde{V}'_{2N} \right) - E \xi p' (1 + \tau) \tilde{V}'_{2L}. \quad (18) \]

We note that the sign of all three terms is negative so that the overall sign is negative as well. As a result the substitution effect between risk management and saving carries over to a situation where the use of the two instruments involves a multiplicative risk, i.e., the risk of non-reliability or an uncertain return. The reason is that this additional source of uncertainty affects, of course, the optimal use of each instrument but does not change their interaction.

4.5 Non-separable Utility

Thus far we have assumed that utility is intertemporally separable, i.e., changes in consumption at one point in time do not affect marginal utility of consumption at other points in time. To avoid confusion, we denote by lower-case \( u = u(c_1, c_2) \) utility of an intertemporal consumption bundle \((c_1, c_2)\) when this assumption is relaxed. Then, the DM’s objective is given by

\[
\max_{e,s} EU = p(e)u(w_1 - c(e) - s, w_2 - l(e) + (1+r)s) + (1 - p(e))u(w_1 - c(e) - s, w_2 + (1+r)s), \quad (19)
\]

and we obtain the relevant cross-derivative of expected utility with respect to risk management and saving as follows:

\[
EU_{es} = c' (pu_{11}(L) + (1 - p)u_{11}(N)) - c' (1 + r) (pu_{12}(L) + (1 - p)u_{12}(N)) + p' [u_1(N) - u_1(L)] + p' (1 + r) [u_2(L) - u_2(N)] + pl' u_{12}(L) - pl' (1 + r) u_{22}(L). \quad (20)
\]

Subscripts denote partial derivatives with respect to consumption, “L” denotes the consumption bundle associated with the second-period loss state and “N” denotes the consumption bundle associated with the second-period no-loss state. The first term is negative due to first-period risk aversion, the fourth one is negative due to second-period risk aversion and the sixth one is also negative due to second-period risk aversion. The signs of the second, third and fifth term depend on the cross-derivative of utility with respect to consumption today and tomorrow. They are negative if the DM is correlation-loving \((u_{12} > 0)\) or correlation-neutral \((u_{12} = 0)\), which corresponds to the intertemporally separable case (Eeckhoudt et al., 2007). Note that also the assumption of correlation aversion \((u_{12} < 0)\) is not incompatible with the behavioral
notion of substitution between risk management and saving as long as the individual is “not too” correlation-averse.\textsuperscript{23}

5 Conclusion and Discussion

This paper studies the interplay of intertemporal risk management and saving decisions. Inspired by Lee (1998), we interpret risk management broadly. In our model, it may take the form of reducing the probability of loss, the severity of loss, or both simultaneously. Although theoretical research has often analyzed self-protection and self-insurance separately, due to fundamental differences between these two instruments as related to market insurance, comparative risk aversion, and income effects, real-life examples suggest that often both mechanisms are at work at the same time. Moreover, we explicitly model the intertemporal cost-benefit structure inherent in many risk management decisions. To this end, we assume expenditures to precede the benefit in time. As soon as this precedence is taken into account, saving appears as a natural alternative to risk management. The reason is the apparent similarity in its cost-benefit structure, which is to trade-off current consumption for expected future consumption.

This intuition carries over to the formal analysis. In the benchmark model with separable utility and two states of the world we find a substitution effect between risk management and saving which extends prior literature (Dionne and Eeckhoudt, 1984; Menegatti and Rebessi, 2011). We show that this finding appears as a universal property of joint risk management and saving decisions. It carries over to cases where multiple states of the world are considered, where the effects of risk management are not restricted to first-order stochastic changes in the distribution of final wealth, to cases of instrument non-reliability and uncertain returns, and to situations in which utility is no longer intertemporally separable as long as correlation aversion is moderate. In this sense, the substitution effect between risk management and saving is a very robust phenomenon that appears in a huge variety of potential model extensions. We study its consequences in the context of changes in income and background risk. The intuition is that, besides direct effects, substitution between the two instruments introduces an indirect effect that is typically of opposite sign as the direct one. Consequently, all exogenous changes will have ambiguous marginal effects on the instruments at hand and conditions to resolve this ambiguity depend much on the structure of the decision problem at hand.

\textsuperscript{23} Technically speaking, correlation-loving and correlation-neutral preferences are sufficient for substitution between risk management and saving, whereas correlation aversion is necessary when there is complementarity between the two instruments.
Our paper highlights the fundamental difference between single- and multi-instrument models. Whereas income effects and changes in background risk have clearcut effects if risk management and saving decisions are considered in isolation, this is no longer true when their interaction becomes relevant. As such empirical work needs to identify whether individuals take integrated decisions in optimizing their intertemporal consumption pattern or not. Without such identification it is not clear how to interpret aggregate data or whether interpretation is possible at all. Second, as soon as various instruments are jointly in use, individual heterogeneity in risk and time preferences, heterogeneity in the properties of the various instruments, and heterogeneity in exogenous income levels will determine whether substitution prevails or not as a reaction to exogenous changes. Consequently, a null response in one instrument does not necessarily imply that individuals do not react to incentives properly or that their behavior is inconsistent with theory. As long as we cannot confidently rule out that negative substitution effects from other instruments even out direct effects, it is hard to draw an ultimate conclusion.

This paper is devoted to two important instruments which DMs use to cope with future consumption uncertainty, risk management and saving. Substitution between the two is a fundamental property with far-reaching consequences for all kinds of comparative statics analysis. We hope that our paper provides guidance in how to implement these cross-effects in future theoretical and empirical work. This can help achieve a more complete picture of all types of behavioral reactions to exogenous variation.
References


Appendix

Proof of Proposition 1: By Footnote 7, the second-order condition is globally satisfied, i.e., $EU_{ee} < 0$. Therefore, by application of the implicit function theorem to (3), the direction of a marginal effect regarding an exogenous parameter is given by the respective cross-derivative. Now

$$EU_{ew_1} = -c'U''_1 > 0,$$
$$EU_{ew_2} = \beta p'(V''_2L - V''_2N) - \beta p'l'V''_2L < 0,$$
$$EU_{e\beta} = p'(V_2L - V_2N) - pl'V''_2L > 0,$$

which completes the proof. □

Proof of Remark 1: Differentiating the first-order condition (5) with respect to saving yields

$$\frac{de}{ds} = -\frac{EU_{es}}{EU_{ee}} = -\frac{c'U''_1 + \beta p'(1 + r)[V''_2L - V''_2N] - pl'V''_2L(1 + r)}{EU_{ee}}.$$

It is easy to see that $EU_{es}$ is negative and so is $de/ds$. □

Proof of Proposition 2: Totally differentiating the system of the first-order conditions (5) and (6) with respect to $w_1$ yields

$$EU_{ee}de + EU_{es}ds + EU_{ew_1}dw_1 = 0,$$
$$EU_{ss}ds + EU_{es}de + EU_{sw_1}dw_1 = 0.$$

From this we can solve for the marginal effects on risk management and saving,

$$\frac{de}{dw_1} = -\frac{1}{D} (EU_{ss}EU_{ew_1} - EU_{es}EU_{sw_1}),$$
$$\frac{ds}{dw_1} = -\frac{1}{D} (EU_{ee}EU_{sw_1} - EU_{es}EU_{ew_1}),$$

where $D$ is the determinant of the Hessian, $D = EU_{ee}EU_{ss} - EU_{es}^2$. It is assumed to be positive for maximality. We can now rewrite the effects in terms of marginal benefits and costs of the different activities. Note that $EU_{ew_1} = -MC_{w_1}^e = -c'U''_1 = MC_s^e$ and that $EU_{sw_1} = -MC_{w_1}^s = -U''_1 = MC_s^s$. Furthermore, as both risk management and saving imply costs only in the first
period, it holds that $MC^e = c'U'_1 = c'MC^s$. With these relations we can simplify

$$-EU_{es}EU_{ew} + EU_{es}EU_{ew} = -MC^e_s (-MC^e_s + MB^e_s) + MC^e_s (-MC^e_s + MB^e_s)$$

$$= MC^e_s (MB^e_s - c'MB^e_s),$$

from which we obtain condition (7), and

$$-EU_{es}EU_{ew} + EU_{es}EU_{ew} = -MC^e_s (-MC^e_e + MB^e_e) + MC^e_s (-MC^e_s + MB^e_s)$$

$$= MC^e_s (c' (MB^e_s - MC^e_s) - (MB^e_e - MC^e_e)),$$

from which we obtain condition (8). Consequently, we can conclude that $de/dw_1$ is positive if and only if (7) holds and that $ds/dw_1$ is positive if and only if (8) holds. The procedure for the other exogenous variables $w_2$ and $\beta$ is analogous.

**Proof of Remark 2:** The idea of the proof is to evaluate the first-order expression in the presence of background risk at the optimal intensity of risk management when background risk is absent. Due to concavity of the objective function, a positive (negative) sign indicates that the introduction of background risk implies an increase (decrease) of optimal risk management.

For FPBR we obtain

$$-c'E\tilde{U}'_1 + \beta p'[\tilde{V}_2L - \tilde{V}_2N] - \beta pl'\tilde{V}'_2L = -c' \left( \mathbb{E}\tilde{U}'_1 - U'_1 \right),$$

due to the first-order condition (3) without background risk. For a prudent DM marginal utility is convex and consequently $\mathbb{E}\tilde{U}'_1 > U'_1$ by Jensen’s inequality. Overall the sign is negative indicating a decrease in risk management.

For SPBR we obtain

$$-c'EU'_1 + \beta p'[\mathbb{E}\tilde{V}_2L - \mathbb{E}\tilde{V}_2N] - \beta pl'\mathbb{E}\tilde{V}'_2L$$

$$= -\beta p' \left[ \mathbb{E} \left( \tilde{V}_2N - \tilde{V}_2L \right) - (V_2N - V_2L) \right] - \beta pl' \left( \mathbb{E}\tilde{V}'_2L - V_2L \right).$$

For a prudent DM, the second summand is positive according to Jensen’s inequality. For the first summand observe that $V_2N - V_2L$ is a convex function of wealth when preferences exhibit prudence. Consequently, another application of Jensen’s inequality demonstrates that also the first summand is positive. The net effect is an increase in the risk management activity. □
Proof of Proposition 3: By application of the implicit function rule we have to determine the cross-derivative of expected intertemporal consumption utility with respect to risk management and the parameter reflecting the size of the background risk, $k$. In case of FPBR we obtain that

$$EU_{ek} = -c'\mathbb{E}\tilde{\varepsilon}\tilde{U}_1'' = -c'\text{Cov}\left(\tilde{\varepsilon}, \tilde{U}_1''\right),$$

where $\tilde{U}_1''$ denotes the second derivative of first-period utility in the presence of background risk. When the background risk takes high values, consumption in the first period is high and so is the second derivative of first-period utility due to prudence. Consequently, the covariance is positive rendering the cross-derivative negative. As a result it is optimal to decrease the level of risk management.

For SPBR the relevant cross-derivative is given by

$$EU_{ek} = \beta p'\mathbb{E}\tilde{\varepsilon}\left(\tilde{V}'_{2L} - \tilde{V}'_{2N}\right) - \beta pl'\mathbb{E}\tilde{\varepsilon}\tilde{V}'_{2L} = \beta p'\text{Cov}\left(\tilde{\varepsilon}, \tilde{V}'_{2L} - \tilde{V}'_{2N}\right) - \beta pl'\text{Cov}\left(\tilde{\varepsilon}, \tilde{V}'_{2L}\right).$$

The notation is analogous to before. When the background risk takes high values, consumption in the second period increases. As a result, the difference between marginal utility in the loss state and in the no-loss state shrinks because $V'$ is convex under prudence. Hence, the first covariance term is negative which, together with $p' < 0$, is a positive effect. Secondly, when the background risk takes high values, consumption in the loss state is larger and so is the second derivative of second-period utility due to prudence. As a result, the second covariance term is positive which, together with $-\beta pl' > 0$, is a positive net effect. Overall it is optimal to increase the level of risk management.

Proof of Proposition 4.a: As in the proof of Proposition 2, we utilize the implicit function theorem to obtain

$$\frac{de}{dk} = -\frac{1}{D} (EU_{ss}EU_{ek} - EU_{es}EU_{sk}),$$
$$\frac{ds}{dk} = -\frac{1}{D} (EU_{es}EU_{sk} - EU_{es}EU_{ek}).$$

$D$ is the determinant of the Hessian matrix which is assumed to be positive for maximality. Observe that $EU_{ek} = -\mathbb{E}\tilde{\varepsilon}\tilde{MC}_e^{w_1} = \mathbb{E}\tilde{\varepsilon}\tilde{MC}_e^{a}$ and that $EU_{sk} = -\mathbb{E}\tilde{\varepsilon}\tilde{MC}_s^{w_1} = \mathbb{E}\tilde{\varepsilon}\tilde{MC}_s^{a}$. From this it follows that the effect of background risk on the margin is nil because when $k = 0$ both $\tilde{MC}_e^{a}$ and $\tilde{MC}_s^{a}$ are deterministic such that $EU_{ek} = EU_{sk} = 0$. After some algebraic rearrangements,
we obtain
\[
\begin{align*}
\frac{de}{dk} &= \frac{1}{D} \mathbb{E}\tilde{\varepsilon}\tilde{MC}_s^s (MB_s^e - c' MB_s^e), \\
\frac{ds}{dk} &= \frac{1}{D} \mathbb{E}\tilde{\varepsilon}\tilde{MC}_s^s \left( c'(MB_s^e - \mathbb{E}\tilde{MC}_s^e) - (MB_s^e - \mathbb{E}\tilde{MC}_s^e) \right).
\end{align*}
\]

If the DM is prudent in the first period, then \(-U''\) is a decreasing function. Consequently, for higher realizations of \(\tilde{\varepsilon}\) the corresponding realization of \(\tilde{MC}_s^s\) is lower. As a consequence the two random variables covary negatively so that \(\mathbb{E}\tilde{\varepsilon}\tilde{MC}_s^s = \text{Cov} \left( \tilde{\varepsilon}, \tilde{MC}_s^s \right) < 0\). Hence, the bracketed expressions determine the overall sign of the marginal effects.

\textbf{Proof of Proposition 4.b:} This is very similar to the proof of Proposition 4.a and will be omitted.